Problem Set: Balanced Growth, Non-Balanced Growth and the STraP

(Due on February 20 @ 17pm)

Consider the model of structural and economic growth in Ngai and Pissarides (2007), which is a fully dynamic version of the ideas in Baumol (1967).

Households have CRRA preferences over sequences of a consumption aggregate C_t

$$U = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\theta}}{1-\theta}$$

and chooses consumption C_t and investment X_t in a capital K_t whose service rents to firms, together with the services of the fixed labor supply subject to standard period by period budget constraint and law of motion of capital:

$$P_{ct}C_t + P_{xt}X_t + P_{ct}B_{t+1} = W_t L + R_t K_t + (1+r_t) P_{ct} B_t$$

and

$$K_{t+1} = X_t + (1-\delta) K_t.$$

The production side of the economy consists of a CRS, CES technology to produce the consumption aggregate using consumption goods from three sectors: agriculture (a), manufacturing (m), and services (s):

$$C_t = \left[\sum_{i=a,m,s} \omega_i^{\frac{1}{\sigma}} C_{it}^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where σ is the elasticity of substitution. Assume $\sigma < 1$.

The output in each sector is produced with Cobb-Douglas technologies

$$Y_{it} = A_{it} K_{it}^{\alpha} L_{it}^{1-\alpha},$$

with $A_{it} = (1+g_i)^t$, with $g_a > g_m > g_s$.

The ouput of the agriculture and service sector is solely used as inputs for the production of the consumption aggregate, while the output of manufacturing sector is also used for investment. That is, the sectoral market clearing conditions are

$$C_{it} = Y_{it},\ i = a, s$$

and

$$C_{mt} + X_{mt} = Y_{mt}.$$

1- Using the first order condition of the households problem derive an Euler equation relating the growth rate of the consumption aggregate, C_{t+1}/C_t, the growth rate of the relative price of investment, P_{xt+1}/P_{xt}, the growth rate of the relative price of consumption, P_{ct+1}/P_{ct}, the future rental price of capital in units of the investment goods, R_{t+1}/P_{xt+1},and the parameters β, θ, and δ.

2- Using the first order condition of the firms' problems, obtain an expression for the the future rental price of capital in units of the investment goods, R_{t+1}/P_{xt+1} in terms of the productivity of the manufacturing sector, A_{mt+1}, the aggregate capital to labor ration, K_t/L , and the paramter α.

3- Assuming $\theta = 1$ and define detrended variables $\tilde{k}_t = (K_t/L)/A_{mt}^{\frac{1}{1-\alpha}}$, $\tilde{x}_t = (X_t/L)/A_{mt}^{\frac{1}{1-\alpha}}$, $\tilde{e}_t = (E_t/L)/A_{mt}^{\frac{1}{1-\alpha}}$, where $E_t = P_{ct}C_t/P_{xt}$ is consumption expenditure in units of the investment good. Solve for the balance growth path of the system in term of these detrended variables.

4- Obtain an express for the time path of the growth rate of the consumption aggregate, C_{t+1}/C_t , and the real interest rate, r_t. along a balance growth path.

5- Assume $\theta \neq 1$. Obtain expressions for the value of detrended capital $\tilde{k}_t$ in the asymptotic balance growth paths as $t \to -\infty$ and $t \to \infty$, $\tilde{k}_{-\infty}$ and $\tilde{k}_{\infty}$.

Assume the following value for the parameters: $\sigma = 0.5$, $\beta = 0.98$, $\alpha = 1/3$, $g_a = 0.02$, $g_m = .01$, $g_s = 0.005$, $\delta = 0.06$, and $\theta = 2$. You can set $\omega_i = 1$ for all sectors.

6- Solve for the STraP in the model in this case. In particular, starting from $t = -300$ and $\tilde{k}_{-300} = \tilde{k}_{-\infty}$ solve for the transitional dynamic of the model (which will eventually converge towards the asymptotice BGP $\tilde{k}_{\infty}$). You can use a standard shooting algoritm, where you iterate over intial expenditure $\tilde{e}_{-300}$ in order for the value of detrended capital and expenditure at a late data, e.g., $t = 500$, is closed to the asymptotic BGP, i.e., $\left(\tilde{k}_{500}, \tilde{e}_{500}\right) \approx \left(\tilde{k}_{\infty}, \tilde{e}_{\infty}\right)$. (Hint: It will be important to re-shoot along the transition.)

7- Consider now a non-homothetic version of the model, in which the consumption aggregator is now a non-homothetic CES, defined as

$$1 = \sum_i \left(\frac{C_i}{C^{\epsilon_i}}\right)^{\frac{\sigma-1}{\sigma}}.$$

Obtain expressions for the value of detrended capital $\tilde{k}_t$ in the asymptotic balanced growth paths as $t \to -\infty$ and $t \to \infty$, $\tilde{k}_{-\infty}$ and $\tilde{k}_{\infty}$. Notice that in both limits only one sector dominates the economy.

8. Characterize the Euler equation of the system and the intra-period allocation of consumption.

9. Assume the same parameter values as before and $\epsilon_a = 0.2$,$\epsilon_m = 1, \epsilon_s = 1.4$. Characterize the STraP of the system. Compare with the homothetic version of the model.