Structural Transformation: Motivation and Supply-Side Theories

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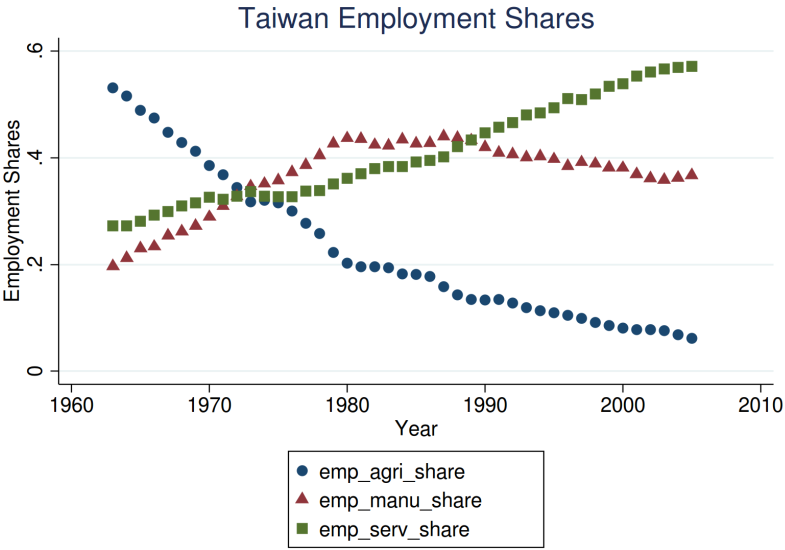
Motivation

- Most modern growth focused on one sector growth model,
  - We have seen some multi-sectoral models, but allocation of factors of production was constant (along BGP)
- It abstracts from structural transformation
  - Yet, reallocation of economic activity is intrinsically related to development.
- Plan
  1. Review "Stylized facts" of structural transformation.
  2. Study multi-sector models that can account for it


Taiwan Employment Shares
Employment Shares
0
.2
.4
.6
1960
1970
1980
1990
2000
2010
Year
emp_agri_share
emp_manu_share
emp_serv_share

Stylized facts

- Structural transformation is defined as the *reallocation of economic activity across three broad sectors: agriculture, manufacturing and services that accompanies the process of modern growth.*
- Three widely used measures:
    1. Employment shares
    2. Value added shares
    3. Final consumption expenditure shares
- 1 and 2 relate to production, 3 to consumption: may differ as final goods can embed intermediates from different sectors.
    - Note potential role for international trade to decouple 3 from 1 and 2

Figure 1: Sectoral Shares of Employment and Value Added – Selected Developed Countries 1800–2000

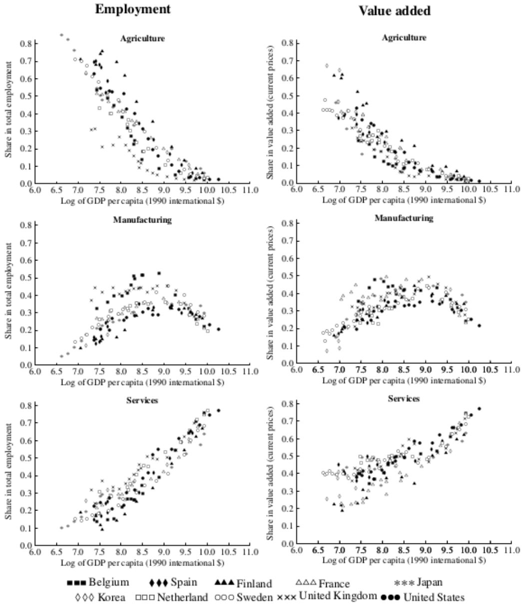


Figure 2: Sectoral Shares of Hours Worked and Nominal Value Added – 5 Non–EU Countries and Aggregate of 15 EU Countries from EU KLEMS 1970–2007

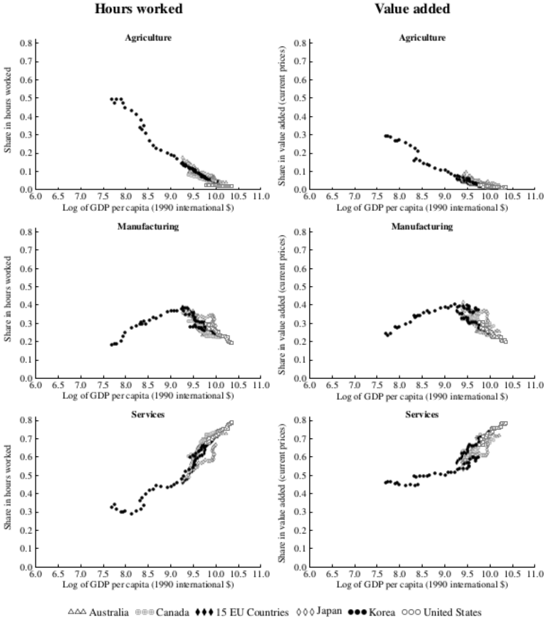


**Figure 5: Sectoral Shares of Employment –
Cross Sections from the WDI 1980–2000**

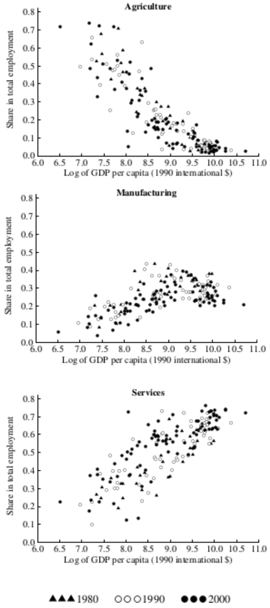


**Figure 6: Sectoral Shares of Nominal Value Added –
Cross Sections from UN National Accounts 1975–2005**

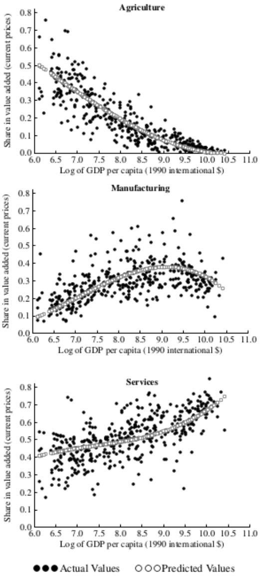


Figure 7: Sectoral Shares of Nominal Consumption Expenditure – US and UK 1900–2008

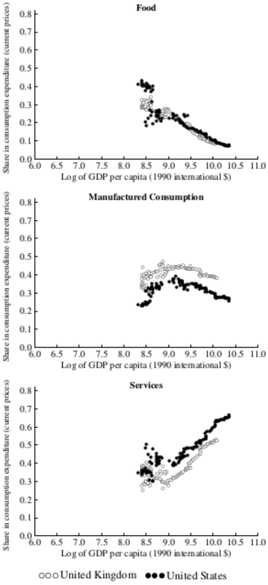
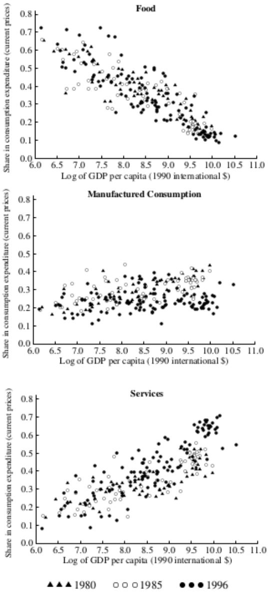


Figure 9: Sectoral Shares of Nominal Consumption Expenditure – Cross Sections from the ICP Benchmark Studies 1980, 1985, 1996



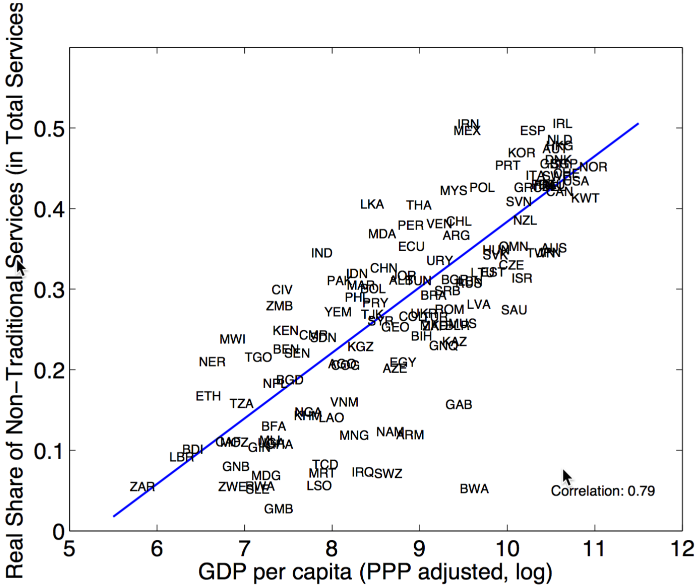
Other Important Stylized Facts

- There exists important structural transformation within services.
  - Crucial for rich economies.
  - Diverging trends between modern vs. traditional service sectors
    - Restuccia and Duarte (2016 WP), Herrendorf et al (2017 WP)
  - "Marketization" of home production & female labor force participation (Buera, Kaboski, . . .).
- Skill-biased technological progress
  - Buera, Kaboski & Rogerson (2016 WP), open economy: Cravino & Sotelo (2017, WP)
- Consumption vs Investment

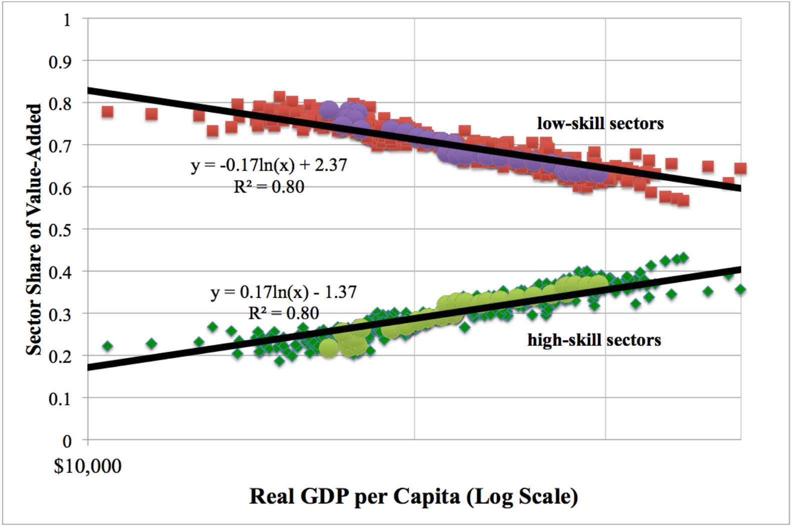
Zooming in Services

- Restuccia and Duarte define
  - Traditional services: gov't, housing, health services, and education
  - Non-traditional services: the rest,e.g., transport services, communication services, and financial and related services.

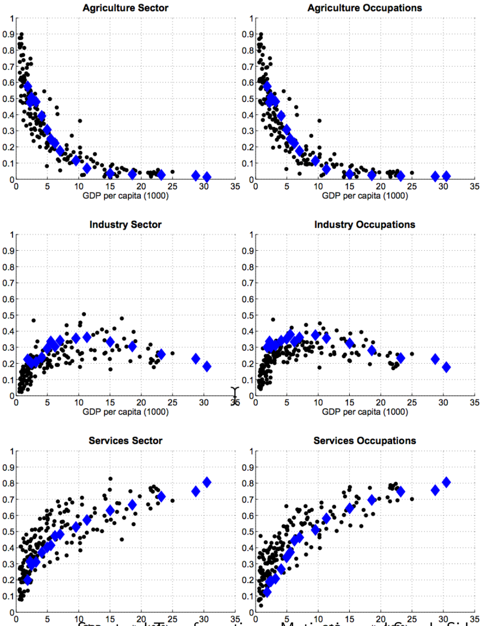
Zooming in Services



Skilled vs. Unskilled Sectors (Buera et al. 2021)



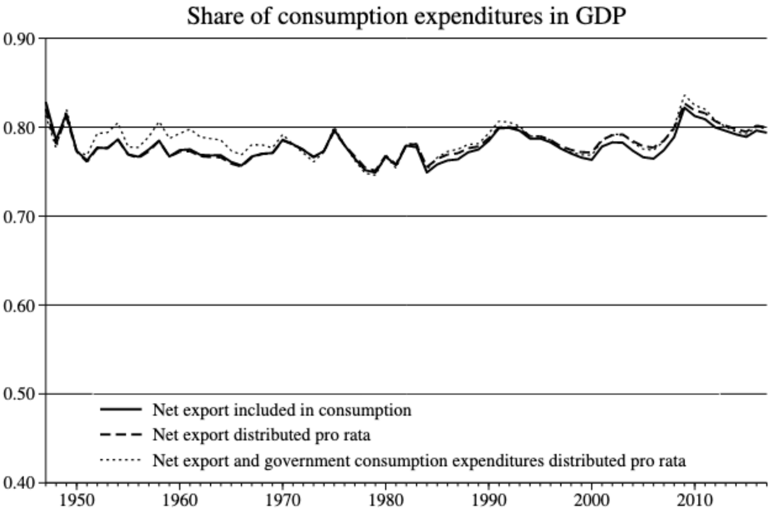
Occupations (Herrendorf and Duernecker 2021)



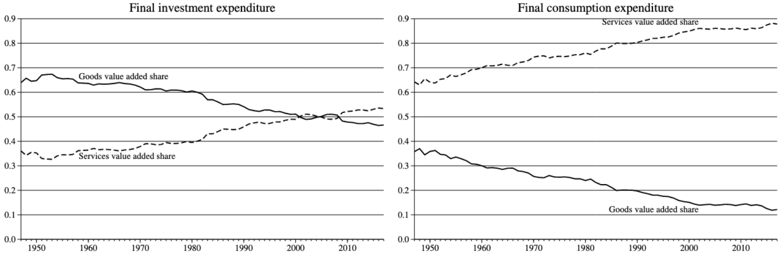
Consumption vs Investment

- Can decompose e in IO calculations between
  - Final Consumption Expenditures
  - Investment
- How do they behave?
- Garcia-Santana, Pi-Joan Mas and Villacorta (21) make the cross-sectional observation that there is SC also in investment
- HRV look at the US time series and confirm it.
- Garcia-Santana et al. (21) show (also documented by others) that savings rate is hump shaped over development path.

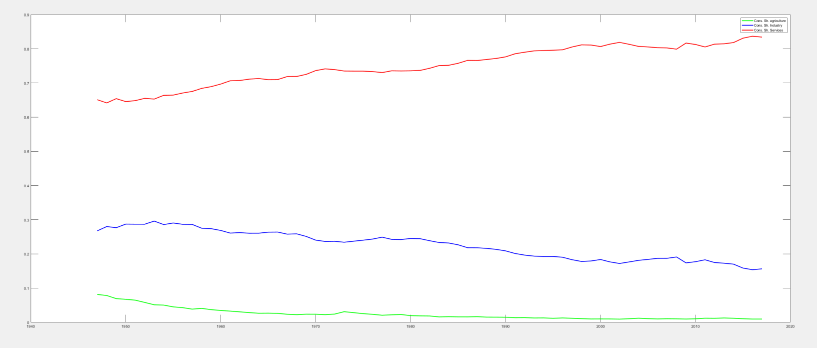
HRV 21: US Consumption Shares



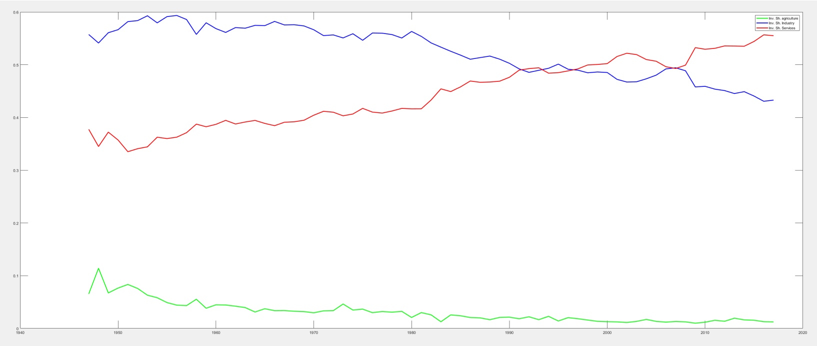
HRV 21 US Consumption and Investment VA Shares



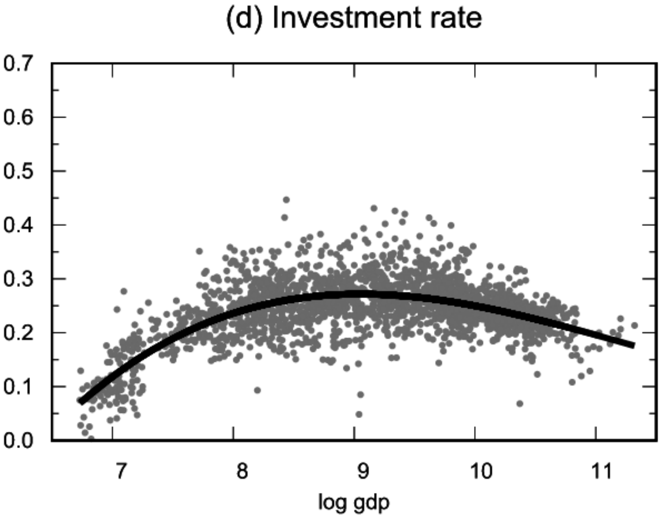
US Consumption VA Shares: 3 Sectors



US Investment VA Shares: 3 Sectors

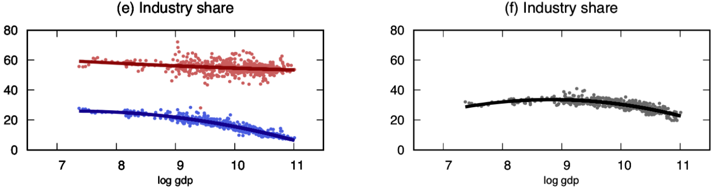


Savings Rate over Development

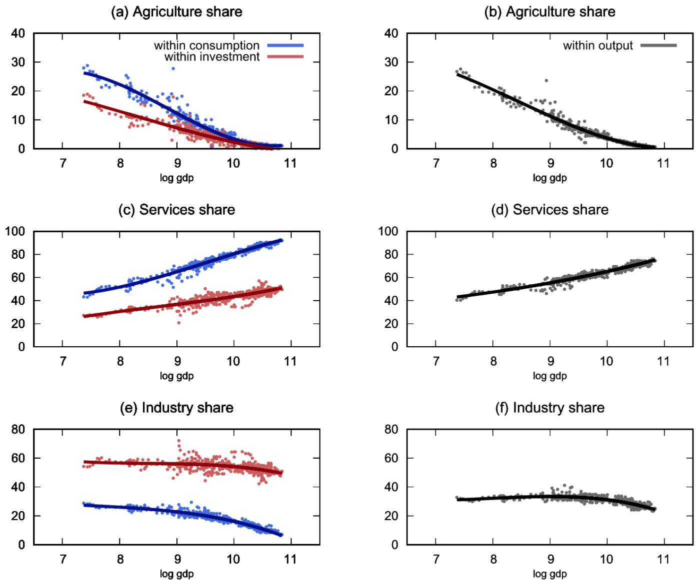


Manufacturing Shares 2000-2017 Garcia-Santana et al

WIOD Data



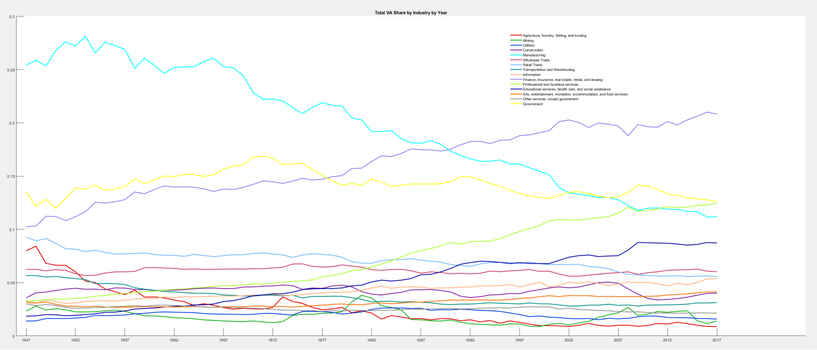
Sectoral Shares 2000-2017 Garcia-Santana et al



Comments

- Is there important structural change in the IO linkages?
  - How does it matter over the development/growth process?
  - The geography services for final consumption and services may be different
- There is clear scope to go beyond 3 sectors to analyze many issues.

Beyond 3 Secors: Total VA Shares for 15 Sectors



Two Views on Drivers of Structural Change

Two Views on Drivers of Structural Change

Supply

Demand

Two Views on Drivers of Structural Change

Trends in Productivity, K-shares

Demand

Two Views on Drivers of Structural Change

Trends in Productivity, K-shares

Non-homothetic Engel Curves

Two Views on Drivers of Structural Change

Trends in Productivity, K-shares		Non-homothetic Engel Curves

Benchmark Model of Structural Change

- Follow exposition in Buera et al. (2020) STraP paper.
- Consider an economy consisting of three sectors:
  - Agriculture (a), manufacturing (m) and services (s).
  - Theory true for arbitrary number of sectors, not only 3.
- Assume a representative consumer and a closed economy.
  - Assume inelastic labor supply
  - Representative agent rules out inequality.
- Output of three sectors used to create two aggregates:
  1. consumption C,
  2. investment X.
- Production in each sector uses capital and labor, although potentially in different proportions.

Representative Agent Problem

- Rep. Agent maximizes:

$$\max_{C(t),X(t),K(t),B(t)} \int_{t=\tau}^{\infty} e^{-\rho(t-\tau)} U(C(C_a(t), C_m(t), C_s(t)))dt,$$

s.t.

$$P_c(t) C(t) + P_x(t) X(t) + P_c(t) \dot{B}(t) = W(t) L + R(t) K(t) + r(t) P_c(t) B(t),$$

and

$$\dot{K}(t) = X(t) - \delta K(t).$$

- Nests benchmark cases, e.g., Ngai Pissarides, Acemoglu-Guerrieri

Model: Household Intratemporal problem

- Household chooses consumption of value-added from
  1. agriculture, $C_a(t)$,
  2. manufacturing, $C_m(t)$,
  3. services, $C_s(t)$,

  to produce period t consumption aggregate

$$C(C_a(t), C_m(t), C_s(t)).$$

- For concreteness assume homothetic CES:

$$C(C_a(t), C_m(t), C_s(t)) = \left[\sum_{j=a,m,s} \omega_{cj}^{\frac{1}{\sigma_c}} C_j(t)^{\frac{\sigma_c - 1}{\sigma_c}} \right]^{\frac{\sigma_c}{\sigma_c - 1}},$$

- Discuss later nonhomohtetic aggregators.
- Assume that sectors are gross complements, i.e. $\sigma_c < 1$.

Model: Investment Sector

- Competitive firm combines sectoral value-added from
  1. agriculture $X_a(t)$,
  2. manufacturing, $X_m(t)$,
  3. and services $X_s(t)$,

  to produce period t investment aggregate.
- For concreteness assume homothetic CES:

$$X\left(t\right)=A_{x}\left(t\right)\left[\sum_{j=a,m,s}\omega_{xj}^{\frac{1}{\sigma_{x}}}X_{j}\left(t\right)^{\frac{\sigma_{x}-1}{\sigma_{x}}}\right]^{\frac{\sigma_{x}}{\sigma_{x}-1}}.$$

- It could be nonhomothetic as well or a different aggregator.
- Note weights ω_{xj} are specific to the investment sector.
- Sector Hicks-neutral technological change $A_{x,t}$,

$$\dot{A}_{x}\left(t\right)=\gamma_{x}(t)A_{x}\left(t\right), \qquad \text{with } \gamma_{x}(t)>0.$$

- Note parameters, ω_{xj}, $\sigma_x \leq 1$, and γ_x all investment-specific.

Model: Sectoral Production

- Competitive CRS representative firm in each sector $j \in \{a, m, s\}$

$$C_j(t) + X_j(t) = A_j F_j(K_j(t), L_j(t)).$$

- $A_j(t)$ Hicks neutral tech.: differential growth by sector.

$$\dot{A}_j(t) = \gamma_j(t) A_j(t),$$

- Differential productivity growth: $\gamma_a > \gamma_m > \gamma_s > 0$.

Model: Feasibility

- labor and capital used by each sector be less than the aggregate supply:

$$\sum_{j=a,m,s} L_j(t) \leq L$$

and

$$\sum_{j=a,m,s} K_j(t) \leq K(t).$$

Equilibrium - Intratemporal Conditions

- Given total amount spent in consumption C

$$C_j = \omega_{jc} \left(\frac{P_j}{P_c} \right)^{-\sigma_c} C$$

where P_C is the ideal price index $P_c = \left(\sum_i \omega_{ic} P_i^{1-\sigma_c} \right)^{1/(1-\sigma_c)}$.

- Analogously, for total investment X

$$X_j = \omega_{jx} \left(\frac{P_j}{P_x} \right)^{-\sigma_X} X$$

- Sectoral prices are given by

$$P_j = \frac{\mathcal{C}_j(W, R)}{A_j},$$

where W and R denote the price of labor and capital, and $\mathcal{C}_j(\cdot)$, the unit cost function associated with F_j.

Equilibrium - Intertemporal Conditions

1) Euler Equation:

$$\theta \frac{\dot{C}(t)}{C(t)} = r(t) - \rho = \frac{R(t)}{P_x(t)} - \delta - \rho + \left(\frac{\dot{P}_x}{P_x} - \frac{\dot{P}_c}{P_c} \right),$$

- where $\theta = -\frac{\frac{\partial^2 U(C)}{dC^2} C}{\frac{\partial U(C)}{\partial C}}$. The interest rate involves:
  - the growth rate of relative price of investment, P_x/P_c
  - the rental rate of capital *in terms of investment*.

2) Law of Motion for Capital:

$$\frac{\dot{K}(t)}{K(t)} = \frac{X(t)}{K(t)} - \delta$$

Computing the Equilibrium

- How to proceed? Does this model generate structural change?

Computing the Equilibrium

- How to proceed? Does this model generate structural change?
- One option: given some initial K, solve CE numerically (shooting).
  - Does not provide much insight on role of model elements.
  - Combines standard neoclassical (capital accumulation effects) of NGM with elements of structural change–hard to assess the contribution of each.
- Appealing alternative: perhaps the model generates BGP?
  - If so, we could drastically simplify model...
  - ...cannot do this without imposing further assumptions.
- Alternative concept to BGP: Stable Transformation Path
  - Isolates structural change dynamics from neoclassical capital accumulation.
  - Allows to assess structural change properties generated by different model assumptions.

Start Focusing on BGP

- Define BGP as an equilibrium path along which aggregate variables measured in the same units (e.g., C, X) grow at constant though potentially different rates.
- Find different sets of sufficient conditions for BGP holding.
  - Some correspond to celebrated papers: e.g., Ngai Pissarides 2007, Rebelo et al 2008.
  - Others not fully explored (to the best of my knowledge).
- Under "realistic" assumptions, BGP fails generically.
  - However, there are cases in which quantitative analysis for US suggests BGP is quite a good approximation (HRV 21).

Requirements for BGP from L.o.M. for K

1) Law of motion for capital:

$$\frac{\dot{K}(t)}{K(t)} = \frac{X(t)}{K(t)} - \delta = \frac{P(t)\,Y(t)}{P_x(t)\,K(t)} - \frac{P_c(t)\,C(t)}{P_x(t)\,K(t)} - \delta.$$

where we use the national accounting identity $PY = P_c C + P_x X$

- balance growth requires common, constant growth of
  - real investment X_t and capital K_t
  - output and consumption expenditures *when translated into units of the investment good*, PY/P_x and $P_c C/P_x$
- constant growth in output and consumption expenditures *when translated into units of the investment good*
- not real consumption C_t that grows at a constant rate, but consumption *expenditures* (in units of investment).

Equilibrium: Requirements for BGP from Euler Equation

2) Euler Equation:

$$\theta \frac{\dot{C}(t)}{C(t)} = r(t) - \rho = \frac{R(t)}{P_x(t)} - \delta - \rho + \left(\frac{\dot{P}_x}{P_x} - \frac{\dot{P}_c}{P_c} \right),$$

- Define $\tilde{C}(t) \equiv P_c(t) C(t) / P_x(t)$, Euler eq. becomes

$$\theta \frac{\dot{\tilde{C}}(t)}{\tilde{C}(t)} = \frac{R_{t+1}}{P_{x,t+1}} - \delta - \rho + (1 - \theta) \left[\frac{\dot{P}_x}{P_x} - \frac{\dot{P}_c}{P_c} \right].$$

- Sufficient conditions for constant growth in $\tilde{C}_t$
  1. a constant rental price of capital in units of investment and
  2. either (i) log intertemporal preferences, i.e., $\theta = 1$, or (ii) constant growth in the relative price of investment *and* constant θ.

Can the evolution of relative price of investment generate BGP?

- By cost minimization, the relative price of investment is then:

$$\frac{P_x(t)}{P_c(t)} = \frac{1}{A_x(t)} \frac{\left[\sum_{j=1}^{J} \omega_{xj} P_j(t)^{1-\sigma_x}\right]^{\frac{1}{1-\sigma_x}}}{\left[\sum_{j=1}^{J} \omega_{cj} P_j(t)^{1-\sigma_c}\right]^{\frac{1}{1-\sigma_c}}}.$$

- Even if R is constant, constant relative price of P_x/P_c imposes stringent conditions on γ_x and γ_i.
- To fix ideas, suppose that γ_x and γ_i are constant, W grows at constant rate. Still have time-varying growth in P_x/P_c if
  1. the CES weights ω differ across consumption and investment;
  2. the elasticities σ differ across consumption and investment.

$\Rightarrow$ Realistic X and C sectors make very hard to get BGP!

Ngai and Pissarides, AER 2007

- Corresponds to the following particular case of our model:
  - Suppose that $\theta = 1$.
  - Suppose that $F_j = K_j^{\alpha} L_j^{1-\alpha}$.
  - Suppose that investment is done with manufacturing exclusively,
    $$X = X_m.$$
  - Suppose that productivity growth γ_i are constant.
  - Suppose that γ_x is also constant (set it to zero to simplify but not crucial).
- Assuming Cobb Douglas production functions with constant shares aggregated with a CES is extremely convenient because aggregate output admits a representative production function.
  - In this case K_j/L_j are equalized across sectors!

Consumption Aggregator

- One can solve for an pseudo-aggregate production function for the consumption sector that holds in equilibrium:

$$C(t) = \mathcal{A}_c(t) K_c(t)^{\alpha} L_c(t)^{1-\alpha},$$

where

$$\mathcal{A}_c(t) = \left[\sum_{j=a,m,s} \omega_{cj} A_j(t)^{\sigma_c - 1} \right]^{\frac{1}{\sigma_c - 1}}.$$

- Very useful trick in different settings (try to do it yourselves!)
- Wage is the value of the marginal product of labor:

$$W(t) = (1-\alpha) P_x(t) \mathcal{A}_x(t) \left(\frac{K(t)}{L} \right)^{\alpha}.$$

Relative Prices

- Let W and R denote the price of labor and capital.
- Then, since production is done using Cobb Douglas:

$$P_j = \frac{1}{A_j} \left(\frac{W}{1-\alpha} \right)^{1-\alpha} \left(\frac{R}{\alpha} \right)^{\alpha}$$

- The price of investment is simply $P_x = P_m$.
- The price of the consumption aggregator is

$$P_c = \frac{1}{A_x \mathcal{A}_c} \left(\frac{W}{1-\alpha} \right)^{1-\alpha} \left(\frac{R}{\alpha} \right)^{\alpha}$$

where

$$\mathcal{A}_c(t) = \left[\sum_{j=a,m,s} \omega_{cj} A_j(t)^{\sigma_c - 1} \right]^{\frac{1}{\sigma_c - 1}} .$$

BGP in the Ngai Pissarides Framework

- Go back to Dynamic Equations
- Euler equation with $\theta = 1$

$$\theta \frac{\dot{\tilde{C}}(t)}{\tilde{C}(t)} = \frac{R_{t+1}}{P_{x,t+1}} - \delta - \rho + (1-\theta)\left[\frac{\dot{P}_x}{P_x} - \frac{\dot{P}_c}{P_c}\right].$$

BGP in the Ngai Pissarides Framework

- Go back to Dynamic Equations
- Euler equation with $\theta = 1$

$$\frac{\dot{\tilde{C}}(t)}{\tilde{C}(t)} = \frac{R_{t+1}}{P_{x,t+1}} - \delta - \rho.$$

- $\tilde{C}$ grows at constant rate iff $\frac{R_{t+1}}{P_{x,t+1}}$ is constant.
- Is this possible?

BGP in the Ngai Pissarides Framework

- Go back to Dynamic Equations
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$$\frac{\dot{\tilde{C}}(t)}{\tilde{C}(t)} = \frac{R_{t+1}}{P_{x,t+1}} - \delta - \rho.$$

- $\tilde{C}$ grows at constant rate iff $\frac{R_{t+1}}{P_{x,t+1}}$ is constant.
- Is this possible?
- Yes, think about the manufacturing producer FOC

$$P_m \alpha \left(L_m/K_m\right)^{1-\alpha} = R,$$

and use that $P_m = P_x$.

Importance of manufacturing-only investment assumption

- Can the Rental Rate be consistent with BGP?
- Aggregate production function of investment

$$X(t) = \mathcal{A}_x(t) K_x(t)^{\alpha} L_x(t)^{1-\alpha},$$

where

$$\mathcal{A}_x(t) = A_x(t) \left[\sum_{j=a,m,s} \omega_{xj} A_j(t)^{\sigma_x - 1} \right]^{\frac{1}{\sigma_x - 1}}.$$

- The rental rate of capital in units of the investment good:

$$\frac{R(t)}{P_x(t)} = \alpha \mathcal{A}_x(t) \left(\frac{K(t)}{L} \right)^{\alpha - 1}.$$

- BGP requires $\mathcal{A}_{x,t}$ grow at constant rate (since K_t should), but not generally true$\rightarrow$ Assuming only one sector contributes to X solves it!

Structural Change in Ngai Pissarides

- Neat result: Along BGP, there is structural change.
  - We showed along BGP $P_t C_t$ grows at investment rate
- Need different technological progress $\gamma_m, \gamma_s, \gamma_a$.
  - Emphasize importance of *differential technological progress* across sectors.

Equilibrium Allocations

- Relative consumption

$$\frac{C_{at}}{C_{mt}} = \frac{\omega_{ac}}{\omega_{mc}} \left(\frac{A_{at}}{A_{mt}} \right)^{\sigma_c}$$
$$\frac{C_{st}}{C_{mt}} = \frac{\omega_{sc}}{\omega_{mc}} \left(\frac{A_{st}}{A_{mt}} \right)^{\sigma_c}$$

- Defining $n_{it} = L_i/L$ and noting that $C_{it} = K_t^{\alpha} A_{it} n_{it}$

$$\frac{n_a}{n_m} = \frac{P_a C_a}{P_m C_m} = \frac{\omega_{ac}}{\omega_{mc}} \left(\frac{A_{mt}}{A_{at}} \right)^{(1-\sigma_c)}$$
$$\frac{n_s}{n_m} = \frac{P_s C_s}{P_m C_m} = \frac{\omega_{sc}}{\omega_{mc}} \left(\frac{A_{mt}}{A_{st}} \right)^{(1-\sigma_c)}$$

- Clear that we need $\sigma_c \neq 1$ to have structural change

Structural Transformation

- Suppose that

$$\gamma_a > \gamma_m > \gamma_s$$
$$\sigma_c < 1$$

- This implies that employment and consumption and output shares are decreasing for agriculture and increasing for services. Ambiguous for manufacturing
  - Ngai and Pissarides show that it is either hump-shaped or monotonically decreasing.

Merits and Limitations

- Can account for the trends in the three sectors both in employment and nominal expenditures.
- The model cannot account for the behaviour of all real shares, irrespective of production or consumption.
  - CES with $\sigma_c < 1$, nominal and real shares necessarily move in opposite directions
- Ngai-Pissarides has been a very influential paper.
  - Paradigm of differential technological progress across sectors generating SC.
- Within homothetic world, standard prices indices for GDP hold, not true with nonhomothetic preferences.

Other Sources of Changes in Relative Prices

- Let's go back to our baseline model.
- We have seen that the relative price of two sectors is given by

$$\frac{P_i}{P_j} = \frac{A_j}{A_i} \frac{\mathcal{C}_i(W,R)}{\mathcal{C}_j(W,R)}$$

- If sectors only differ in Hicks Neutral Productivity term then we recover Ngai-Pissarides world in which $\frac{P_i}{P_j} = \frac{A_j}{A_i}$.
- There is a venerable tradition of papers that posits production functions as functions of only (equipped) labor

$$Y_j = A_j L_j.$$

- For SC, this formulation has the advantage of thinking about "labor productivity" A_j which is simpler to measure in the data, since one only needs output and workers.
- However, there is ample evidence that $F_j \neq F_i$ (see next slide).

Differences in Sectoral Production Functions

- Very interesting topic, but somewhat outside the scope of the class.
- (Sectoral) aggregate production functions are widely used in macro but not very guided by empirics
  - IO literature stirs away from aggregate production function.
  - In fact, hard to show existence and properties of microfounded aggregate production function.
- Herrendorf, Herrington and Valentinyi (AEJ Macro, 15) and Herrendorf and Valentinyi (RED, 2008) have undertaken the challenge of estimating this aggregate production functions.

Heterogeneity in Sectoral Production Functions

- Herrendorf, Herrington and Valentinyi (AEJ Macro, 15) estimate production functions

$$Y_i = \left(\alpha_i \left(e^{\gamma_{ik} t} K_i\right)^{\frac{\sigma_i}{\sigma_i - 1}} + (1 - \alpha_i) \left(e^{\gamma_{il} t} L_i\right)^{\frac{\sigma_i}{\sigma_i - 1}}\right)^{\frac{\sigma_i - 1}{\sigma_i}}$$

$$Y_i = \left(e^{\gamma_{ik} t} K_i\right)^{\alpha_i} \left(e^{\gamma_{il} t} L_i\right)^{1 - \alpha_i}$$

- They document substantial heterogeneity across sectors, for the post WWII US.

Cobb Douglas Sectoral Production Functions (HV 08)

Table 1: Capital income shares at the sectoral level (in producer prices)

Agriculture (A)	0.54
Manufactured consumption (M)	0.40
Services (S)	0.34
Equipment investment (E)	0.34
Construction investment (C)	0.21
Agriculture (A)	0.54
Manufacturing (M+E+C)	0.33
Services (S)	0.34
Consumption (A+M+S)	0.35
Investment (E+C)	0.28
Tradables (A+M+E)	0.37
Nontradables (S+C)	0.32
Agriculture (A)	0.54
Nonagriculture (M+S+E+C)	0.33
GDP (A+M+S+E+C)	0.33

CES Sectoral Production Functions (HHV 15)

Table 1: Estimation Results

	Aggregate	Agriculture	Manufacturing	Services
σ	0.84**	1.58**	0.80**	0.75**
	(0.041)	(0.068)	(0.015)	(0.020)
γ_k	–0.010	0.023**	–0.045**	–0.002
	(0.006)	(0.003)	(0.009)	(0.004)
γ_l	0.022**	0.050**	0.044**	0.016**
	(0.003)	(0.004)	(0.007)	(0.002)
$\bar{\theta}$	0.33	0.61	0.29	0.34

Standard errors in parentheses; ** $p < 0.01$

Sectoral TFP Growth under CD (HHV 15)

Table 2: Average Annual Growth Rates of TFP (in %)

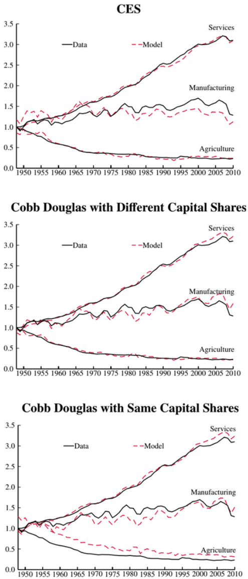
	Aggregate	Agriculture	Manufacturing	Services
CD with α_i	1.1	3.3	1.5	1.0
CD with α	1.1	3.9	1.4	1.0

Herrendorf, Herrington and Valentinyi (AEJ Macro, 15)

- They then develop a structural change model and find that differences in technical progress across the three sectors are the dominant force behind structural transformation whereas other differences across sectoral technology are of second order importance.
- Crucially, they focus on a model with **only intratemporal** effects, but taking the path of total expenditures as given.
- They conclude that Cobb-Douglas sectoral production functions that differ only in technical progress capture the main technological forces behind the postwar US structural transformation.
  - People like this fact (simplifies your life) and they got lots of citations.
  - Not clear the conclusion holds at earlier stages of development.

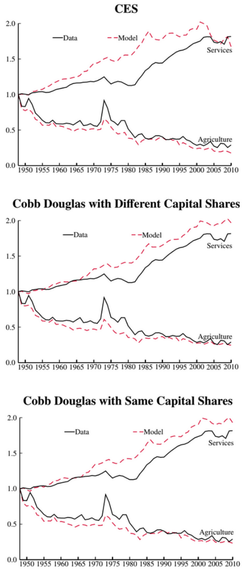
Model Fit: Employment (HHV 15)

Figure 2: Hours Worked (Data=1 in 1948)



Model Fit: Prices (HHV 15)

Figure 3: Sectoral Prices Relative to Manufacturing (Data and Model =1 in 1948)



Diffs in Prod. Functions beyond Hicks Neutral Tech. Prog.

- **Acemolgu and Guerrieri, JPE, 2008**: consider Cobb-Douglas with different Factor Shares

$$Y_i = A_i K_i^{\alpha_i} L_i^{1-\alpha_i}$$

- **Alvarez-Cuadrado, Van Long and Poschke, TE, 2015**: consider CES

$$Y_i = \left(\alpha_i \left(A_i K_i\right)^{\frac{\sigma_i}{\sigma_i - 1}} + \left(1 - \alpha_i\right)\left(B_i L_i\right)^{\frac{\sigma_i}{\sigma_i - 1}}\right)^{\frac{\sigma_i - 1}{\sigma_i}}$$

- Acemoglu-Guerrieri solve a model w/ an intertemporal choice.
- Alvarez-Cuadrado et al. focus only on the intratemporal choices, but show richness of possible outcomes in this setup

Acemoglu Guerrieri (2008): Assumptions

- Suppose capital intensities differ across sectors and tech progress is common across sectors

$$Y_{it} = A_t K_{it}^{\alpha_i} L_{it}^{1-\alpha_i} \quad i \in \{a, m, s\}. \tag{1}$$

- Note: they have two sectors in the paper (here, allow for 3).
  - They also allow for A_{it} but I shut this down here since it is the Ngai Pissarides mechanism and focus on the novel part instead.
- There is no separate investment sector, instead the output from consumption aggregator is used for both consumption and investment:

$$P_x = P_c.$$

- The consumption aggregator is a homothetic CES (as Ngai-P.)
- Assume constant IES (θ).

Equilibrium Conditions: Intratemporal problem

- The capital-labor ratios differ across sectors:

$$\frac{1-\alpha_i}{\alpha_i}\frac{K_{it}}{L_{it}} = \frac{1-\alpha_j}{\alpha_j}\frac{K_{jt}}{L_{jt}}$$

$\Rightarrow$ capital-labor ratios grow at same rate for all sectors.

- Relative prices

$$\frac{P_{it}}{P_{jt}} \propto \left(\frac{K_{it}}{L_{it}}\right)^{\alpha_j-\alpha_i}$$

$\Rightarrow$ the relative price of the more capital intensive good declines (Baumol)

Acemoglu Guerrieri (2008): First Results

- Assume that $\alpha_a > \alpha_m > \alpha_s$
- This generates the "right" pattern of relative prices:
  - P_s/P_m and P_m/P_a both increase over (as Ngai-P) time.
- Provides intuitive justification for $\exists$ capital deepening.
  - Important fact over development (more below).
- Can account for changes in nominal value added shares (if $\sigma_c < 1$) *but fails on real value added*. Recall that

$$\left(\frac{P_{it}}{P_{jt}}\right)^{\sigma_c} \frac{C_{it}}{C_{jt}} = \frac{\omega_a}{\omega_m} = \left(\frac{P_{it}}{P_{jt}}\right)^{\sigma_c - 1} \frac{P_{it} C_{it}}{P_{jt} C_{jt}}$$

Intratemporal Conditions

- Going back to Euler Equation (recall θ is assumed constant)

$$\theta \frac{\dot{\tilde{C}}(t)}{\tilde{C}(t)} = \frac{R}{P_x} - \delta - \rho + (1-\theta)\left[\frac{\dot{P}_x}{P_x} - \frac{\dot{P}_c}{P_c}\right].$$

- Since $P_x = P_c$, last term drops and $\tilde{C} = C$.
- Normalize $P_c = 1$.
- Seems Euler Eq. can be consistent w/ BGP if R is constant,
  - when will R be constant?

Intratemporal Conditions ct'd

- Suppose K, W grows at constant rate and look at MPK of different sectors $MPK_i = \alpha_i A \left(L_i/K_i\right)^{1-\alpha_i}$
- Consider now the problem of the firm in sector i. FOC:

$$P_i \alpha_i A \left(L_i/K_i\right)^{1-\alpha_i} = R \quad where \quad P_i \propto A^{-1} \left(R\right)^{\alpha_i} \left(W\right)^{1-\alpha_i}$$

- But we also now that $K_i/L_i \propto K_j/L_j$, which implies that:

$$R = A \left(R\right)^{\alpha_i} \left(W\right)^{1-\alpha_i} \alpha_i \left(L_i/K_i\right)^{1-\alpha_i}$$

$$R = A \left(\frac{(\alpha_i - 1)\alpha_j}{\alpha_i(\alpha_j - 1)}\right)^{1-\alpha_i} \left(R\right)^{\alpha_j} \left(W\right)^{1-\alpha_j} \alpha_j \left(L_i/K_i\right)^{1-\alpha_j}$$

- Suppose it holds at a point in time, as K_i increases, it does not hold anymore!
- Only has an asymptotic BGP, when a sector dominates the economy and $K_i \simeq K$

Acemoglu Guerrieri 08 Final Remarks

- They calibrate the model to the US using NIPA.
- Show dynamics near asymptotic BGP (aka Constant Growth Path) do not differ very much from BGP.
- AG results rely not only on differences in the sectoral capital intensities, but also on the fact that with Cobb Douglas production functions the elasticity of substitution between capital and labor is equal to one.
- Alvarez-Cuadrado et al. (2015) show that relative prices also depend on the elasticity of substitution between capital and labor.
    - If K and L are perfect substitutes for some sectors they will not use the expensive factor, whereas if it is Leontief for others they will use both no matter what.
    - However, they show that pretty much anything goes, not exactly clear (to me) for what application this is crucial/makes a difference.

The Investment Channel

- Let's go back to our baseline model and simplify it in a different way to focus on the investment channel.
- Here we are going to follow Herrendorf Rogerson and Valentinyi (2021).
- Assume as in Ngai Pissarides that:

$$Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}$$

- Assume constant IES (but not necessarily 1).
- Assume all productivities γ grow at constant rates.
- Intratemporal model corresponds to the one we discussed setting up the model :)

Intertemporal Conditions for Investment Model

- Euler Equation:

$$\theta \frac{\dot{\tilde{C}}(t)}{\tilde{C}(t)} = \frac{R(t)}{P_x(t)} - \delta - \rho + (1-\theta)\left[\frac{\dot{P}_x}{P_x} - \frac{\dot{P}_c}{P_c}\right].$$

- As discussed, constant growth in $\tilde{C}_t$ requires

  1. a constant rental price of capital in units of investment and
  2. either (i) log intertemporal preferences, i.e., $\theta = 1$, or (ii) constant growth in the relative price of investment

Can evolution relative price of investment generate BGP?

- By cost minimization, the relative price of investment is then:

$$\frac{P_x(t)}{P_c(t)} = \frac{1}{A_x(t)} \frac{\left[\sum_{j=1}^{J} \omega_{xj} P_j(t)^{1-\sigma_x}\right]^{\frac{1}{1-\sigma_x}}}{\left[\sum_{j=1}^{J} \omega_{cj} P_j(t)^{1-\sigma_c}\right]^{\frac{1}{1-\sigma_c}}}.$$

- Since sectoral biased productivity, time-varying growth if
  1. the CES weights ω differ across consumption and investment;
  2. the elasticities σ differ across consumption and investment.
- Not possible generically!

Can the Rental Rate be consistent with BGP?

- Aggregate production function of investment

$$X(t) = \mathcal{A}_x(t) K_x(t)^{\alpha} L_x(t)^{1-\alpha},$$

where

$$\mathcal{A}_x(t) = A_x(t) \left[\sum_{j=a,m,s} \omega_{xj} A_j(t)^{\sigma_x - 1} \right]^{\frac{1}{\sigma_x - 1}}.$$

- The rental rate of capital in units of the investment good:

$$\frac{R(t)}{P_x(t)} = \alpha \mathcal{A}_x(t) \left(\frac{K(t)}{L} \right)^{\alpha - 1}.$$

- BGP requires $\mathcal{A}_{x,t}$ grow at constant rate (since K_t should).
  - Need time varying γ's (precluded by assumption)
  - Only asymptotically when one sector dominates...

Discussion

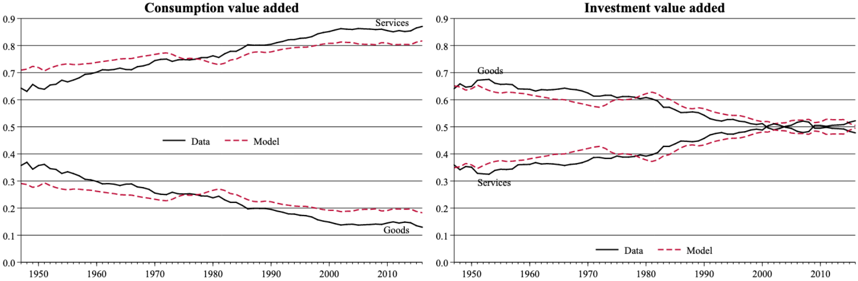
- The model fails to have a BGP.
- If sectors are gross complements in both the C and X aggregators and there is growth, the least productive sector overtakes the economy.
  - HRV 21 argue that this is indeed the case (see calibration numbers next)
- HRV 21 feed in path of expenditure and productivities of the postwar US to a calibrated model (of production functions and preferences)
  - Note that in this case only intratemporal model being tested.
- They show that the model can do well in replicating sectoral behaviour
- Hence, the aggregate growth in the model is also similar to the data (which happens to be close to BGP)

HRV 21 Calibration

Table 1: Preference and technology parameters

ε_c	ω_c	ε_x	ω_x
0.00	0.19	0.00	0.52

HRV 21 Results



Beyond BGP: The Stable Transformation Path

- We saw that except for Ngai Pissarides all other models only have asymptotic BGP.
- How to analyze these models?
  - Simulate model going forward?
  - Stable Transformation Path to isolate Structural Change Dynamics

Competitive Equilibrium of Benchmark Model

Given an initial state consisting of $K(0)$, $A_x(0)$, and $\{A_j(0)\}_{j=a,m,s}$, a **competitive equilibrium** for the model is:

- an allocation, $C(t)$, $K(t)$, $X(t)$, $\{C_j(t), X_j(t), K_j(t), L_j(t)\}_{j=a,m,s}$; and
- prices, $P_c(t)$, $P_x(t)$, $W(t)$, $R(t)$, $r(t)$ and $\{P_j(t)\}_{j=a,m,s}$;

for $t \geq 0$ that solve:

- $B(t) = 0$;
- household optimality and cost-minimizing pricing conditions;
- the transversality condition, $\lim_{t\to\infty} e^{-\rho t} C(t)^{-\theta} K(t) = 0$.

Challenges: little intuition, hard to separate how much initial conditions matter relative to essential ingredients in the model (as we can do with BGP)

Motivating Example: Global Dynamics when $\alpha = \theta$

- For this particular example we can compute the dynamics in closed form.
- Denote $c = C/\mathcal{A}_x^{1/(1-\alpha)}$, $k = K/\mathcal{A}_x^{1/(1-\alpha)}$.
- The solution to the competitive equilibrium given k_0 is:

$$c(t) = M(t)k(t, k_0),$$

$$k(t, k_0) = \left\{ \left[k_0^{1-\alpha} - k^*(t_0)^{1-\alpha} \right] \frac{\mu(t_0)}{\mu(t)} + k^*(t)^{1-\alpha} \right\}^{\frac{1}{1-\alpha}}.$$

- $M(t)$, $k^*(t)$ and $\mu(t)$ are continuous, positively-valued

$$\lim_{t \to \pm\infty} M(t) = \frac{\delta + \rho + (1-\alpha)\gamma_x}{\alpha} - \delta,$$

$$\lim_{t \to \pm\infty} k^*(t) = \overline{k}_{\pm\infty},$$

$$\frac{d\mu(t)}{dt} > 0, \qquad \lim_{t \to \infty} \mu(t) = \infty.$$

Motivating Example: Decomposing the Path of Capital

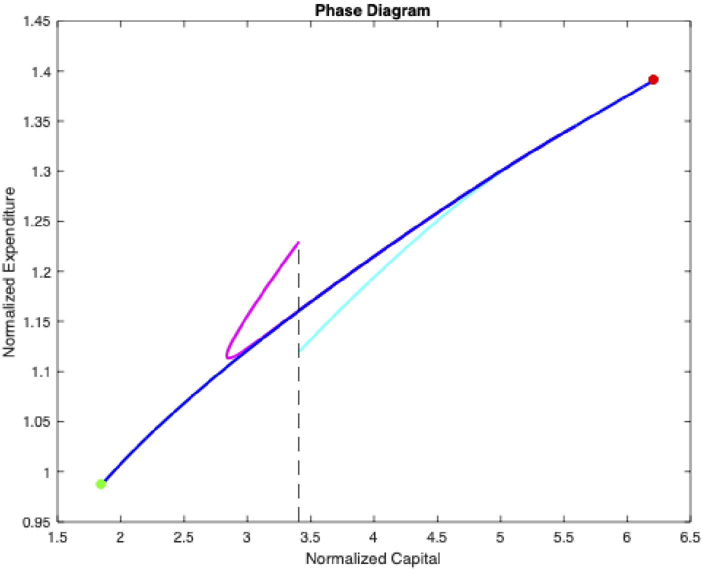
$$k\left(t,k_0\right)=\left\{\underbrace{\left[k_0^{1-\alpha}-k^*\left(t_0\right)^{1-\alpha}\right]}_{\text{Initial Condition}}\underbrace{\frac{\mu\left(t_0\right)}{\mu\left(t\right)}}_{\longrightarrow 0}+\underbrace{k^*\left(t\right)^{1-\alpha}}_{\text{Medium-run}}\right\}^{\frac{1}{1-\alpha}}.$$

- First term vanishes as $t \to \infty$ (exponential decay).
- Medium-run dynamics governed by $k^*(t)$.
- $k^*(t)$ is *independent* of the inital condition k_0.
- $k^*(\infty)$ converges to the asymptotic service-only SS.
- $k^*(-\infty)$ converges to the asymptotic agriculture-only SS.
- $k^*(t)$ is going to be our focus: the "STraP".

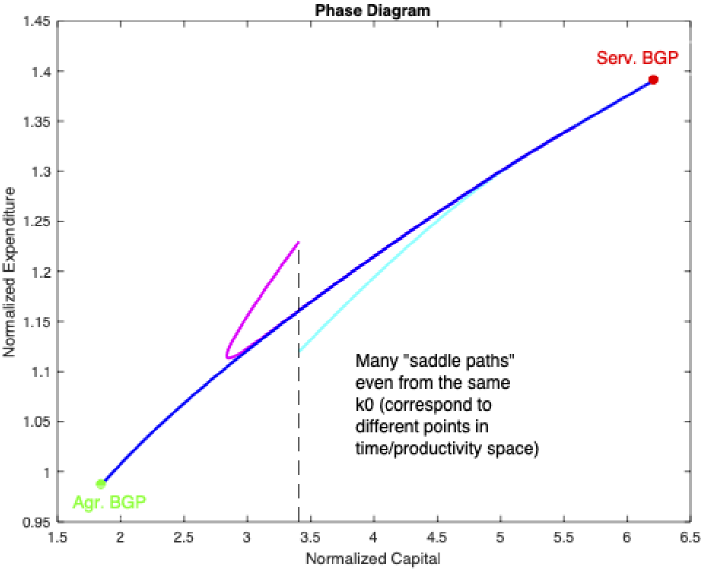
Idea of the STraP

- Observation:
  - The productivity process implies a single sector service economy $t \to \infty$.
  - However, it also implies a single sector agrarian economy as $t \to -\infty$.
  - One can solve analytically for the (normalized) BGP capital values in these two limiting cases.
  - investment productivity is different but constant in these two limiting cases
- can define and solve for constant, investment productivity-normalized capital stocks, $\bar{k}_{-\infty}$ and $\bar{k}_{\infty}$
- For a given productivity process, there exists a unique path linking these two asymptotic BGPs, which is the stable path for this process.

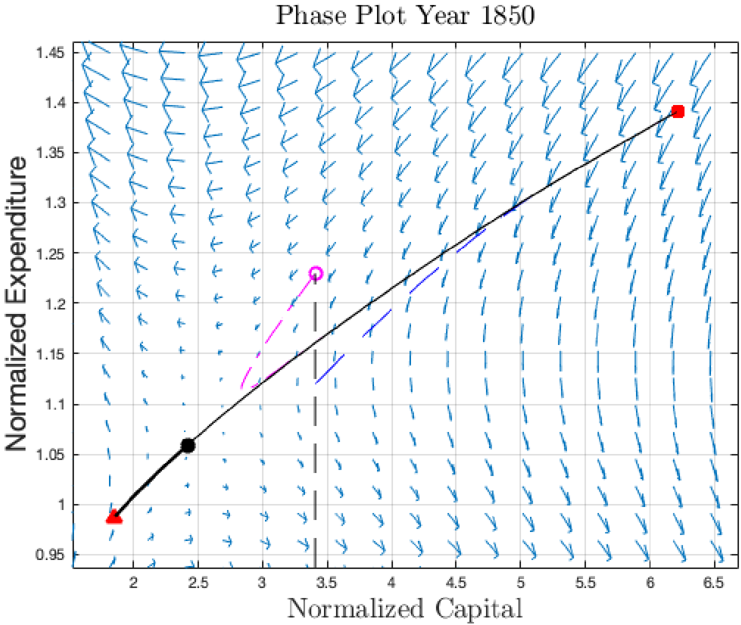
Phase Diagram for k^* and two initial conditions



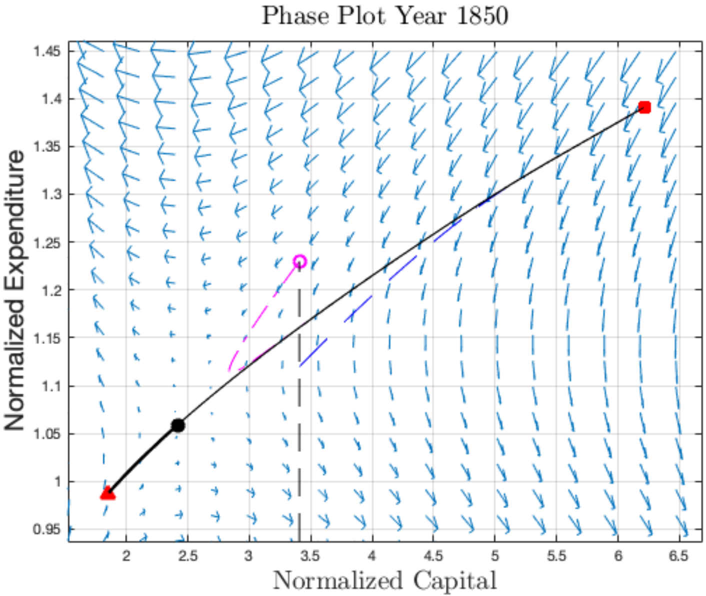
STraP as a "Turn Pike": Phase Diagram



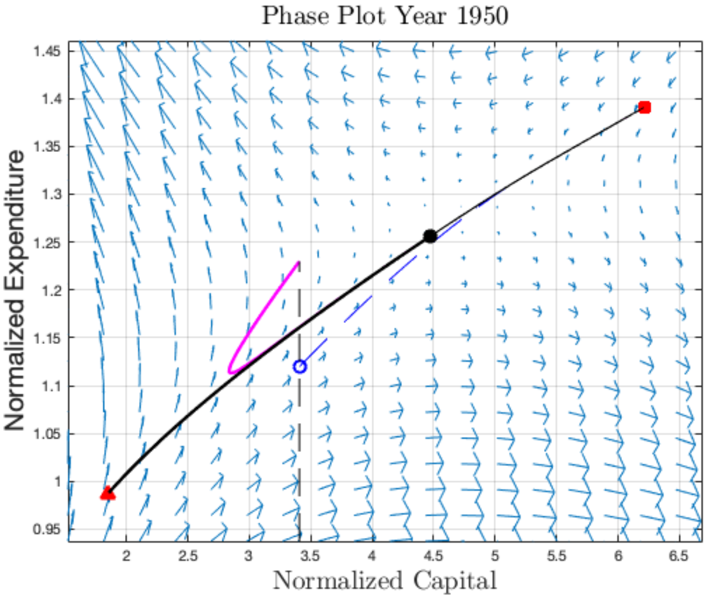
Phase Diagram With Vector Field



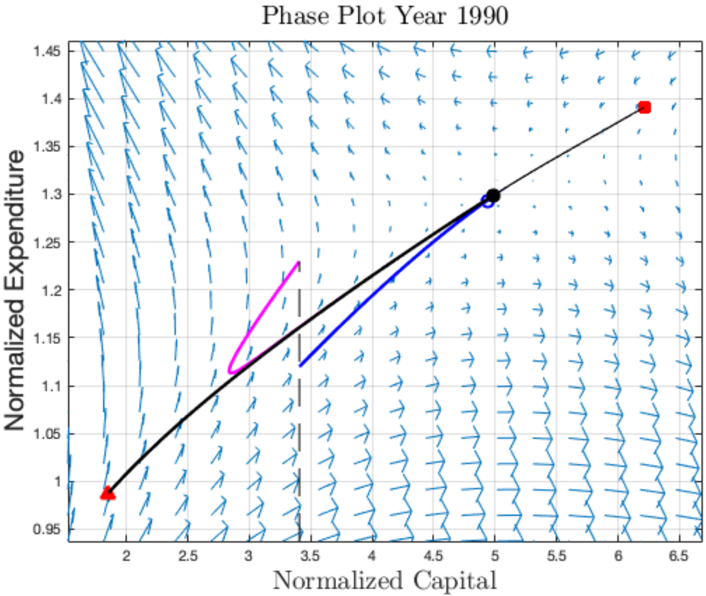
Phase Diagram With Vector Field



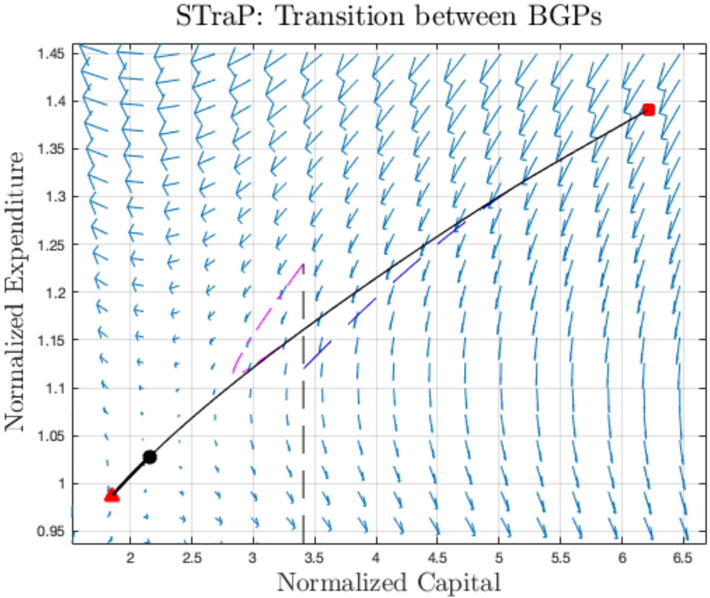
Phase Diagram With Vector Field



Phase Diagram With Vector Field



Phase Diagram With Vector Field - Animation



Defining the STraP in our Model

Given a sequence of productivities, $A_x(t)$, and $\{A_j(t)\}_{j=a,m,s}$, $t \in (-\infty, \infty)$, the **Stable Transformation Path (STraP)** is:

- an allocation, $C(t)$, $K(t)$, $X(t)$, $\{C_j(t), X_j(t), K_j(t), L_j(t)\}_{j=a,m,s}$; and
- prices, $P_c(t)$, $P_x(t)$, $W(t)$, $R(t)$, $r(t)$ and $\{P_j(t)\}_{j=a,m,s}$;

defined $\forall t \in \mathbb{R}$ that solves:

- $B(t) = 0$;
- household optimality and cost-minimizing pricing conditions;
- asymptotic conditions,

$$\lim_{t\to\infty} \frac{K(t)}{\mathcal{A}_x(t)^{1/(1-\alpha)}} = \bar{k}_{\infty},$$

and

$$\lim_{t\to-\infty} \frac{K(t)}{\mathcal{A}_x(t)^{1/(1-\alpha)}} = \bar{k}_{-\infty}.$$

Existence and Uniqueness of STraP: Outline of Proof

- **Thm**: w/regularity condition, **STraP exists and is unique** .
  - Need 2 boundary conditions to solve ODE. Forward is standard.
  - Use existing thm (Hubbard and West 91) for backward part.
  - Have 1D system after solving forward.
  - Construct narrowing upper and lower fences for the system.
  - This generates an "antifunnel" (mild regularity conditions).
  - Solution converges to agriculture BGP as $t \to -\infty$.
- Regularity condition ensures that limit $t \to -\infty$ is well defined. (**Assumption** 1 in the paper).
  - Constructive: limiting properties of production functions, utility and productivity transformation in t.
  - Show it holds in other models (Acemoglu Guerrieri, CLM,. . .).
- We consider a broader class of models than baseline example
  - Allow for nonhomothetic prefs, CES sectoral production.
  - Still require asymptotically model converges to BGP.

Discussion and Differences between CE and STraP

- **The key difference:** STraP replaces initial condition K_0 in the competitive equilibrium with an asymptotic boundary condition, $\overline{k}_{-\infty}$.

$\Rightarrow$ In a competitive equilibrium, K_0 is arbitrary, but the STraP passes through a particular value, K_0^{STraP}.

- K_0^{STraP} is pinned down by the vector of technology.
- $K(t)^{STraP}$ captures medium-term dynamics: the common component of a path that economies starting with different K_0 at $t = 0$ would have.

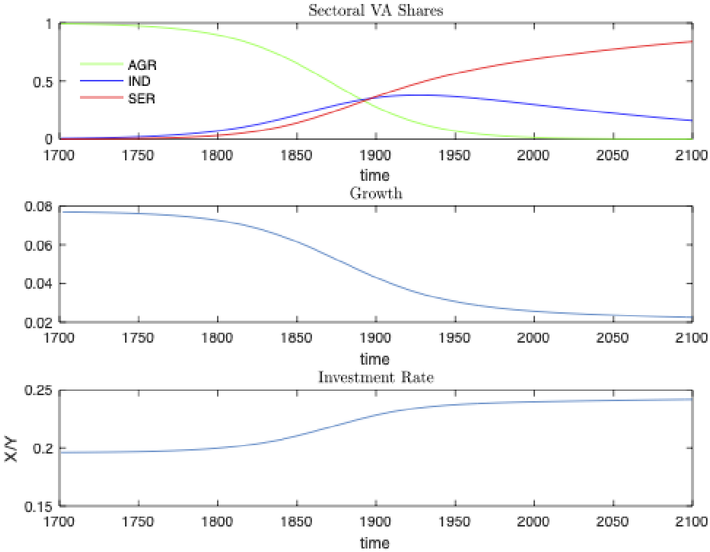
Discussion of STraP

- Key advantage of Strap: allows to evaluate dynamic medium-term properties of models (ie structural change dyanamics)
- Quantitative and qualitative differences from departing from benchmark models e.g. Ngai Pissarides are documented in Buera et al. 2020.

Illustration of STraP: US Calibration post WWII

- Calibrate **Investment** Model using US data post 1947
- Key calibrated parameters:
  - TFP sectoral series $(\{\gamma_j\}, \gamma_x)$ (constant but heterogeneous).
  - C and X CES aggregators: Leontief, $\{\omega_{xj}\} \neq \{\omega_{cj}\}$.
  - IES: $\theta = 2$ (depart from log preferences).
  - Do not need information about the initial capital level!
- Solved with forward-then-backward shooting algorithm.
- Two exercises:
  1. Show importance departures from STraP-enabled benchmark:
     - Log intertemporal preferences, $\theta = 1$, investment rate $\downarrow$ (!).
     - Manufacturing-only investment $\omega_{xm} = 1$, agg. growth $\uparrow$ (!!).
  2. Use model to explore x-country growth dynamics.
     - Use US STraP as a benchmark, compare with x-country data.

Simulation Results (recent U.S. calibration)

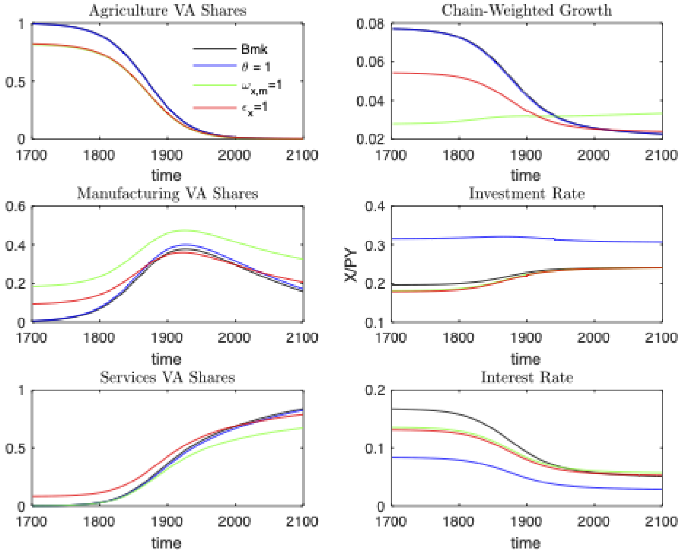


Simulation Results: Alternative Models

Departures from the benchmark model that the STraP enables are important not only quantitatively but qualitatively

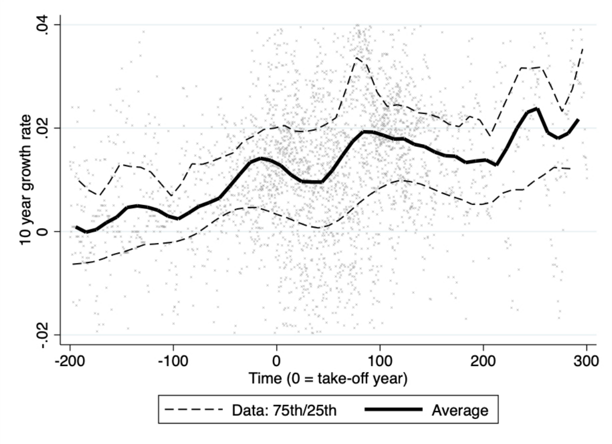
- Impossible without STrAP!!
- Log intertemporal preferences, $\theta = 1$ imply a declining investment rate rather than an increasing one
- Manufacturing-only investment $\omega_{x.m} = 1$ implies an increasing growth rate rather than decreasing growth rate.

Simulation Results: Alternative Models



Growth and Historical Development

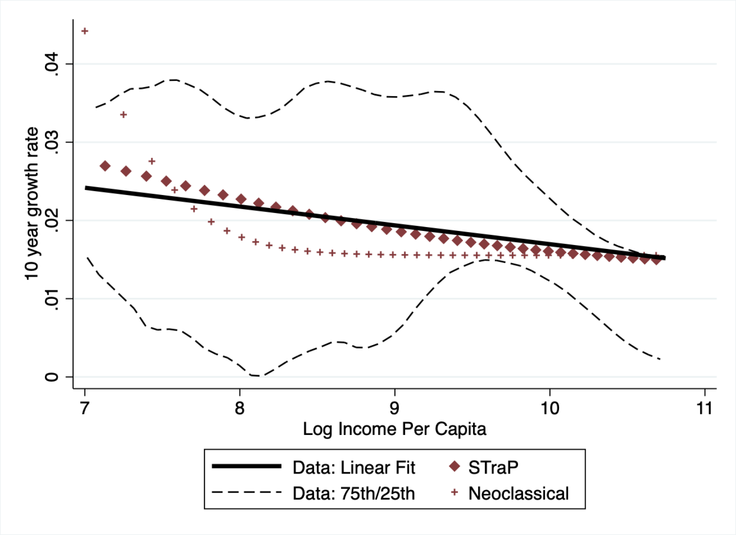
Hist. data (Maddison) for advanced (>UK 1800 income) economies



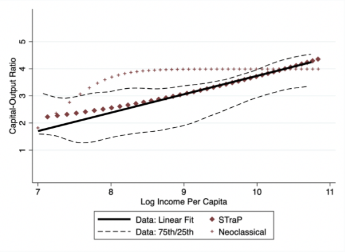
Increasing growth $\implies$ Technology process doesn't hold way back in time!

What about today? 10-Year Growth over Development

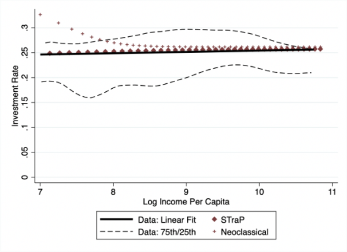
Cross-country Growth Rates post WWII



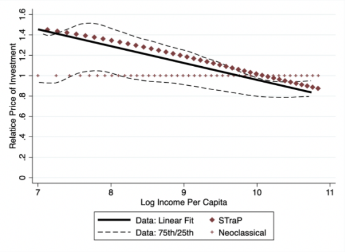
Capital Accumulation and its Determinants



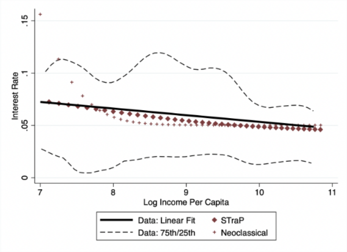
(a) Capital-Output Ratio



(b) Investment Rate



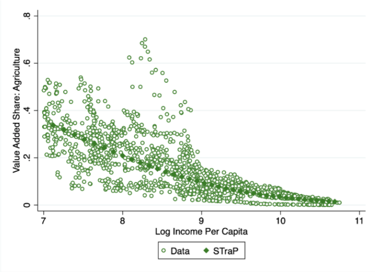
(c) Relative Price of Investment

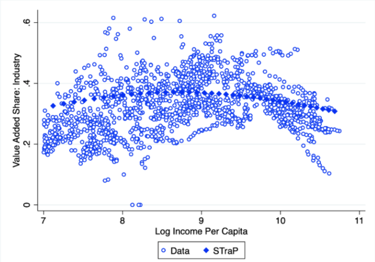


(d) Interest Rate

Structural Change Over Development: Data and STraP

Out of sample test: sectoral data not used in calibration (except US)





STraP allows to separate contribution SC from other mech.

- Can use STraP to separate growth dynamics due to SC from neoclassical capital accumulation.
- Example Growth Decomposition: US vs. Thailand
  - Post-1950 US vs. Thailand comparison
  - US: Data, STraP, CE: 1.6%
  - Thailand: Data: 3.6%, STraP: 2.1%; CE: 2.4%
  - Transition irrelevant for US
  - 12% drop in ag share in Thailand, only 2% drop in US
  - Transition, ST, and high productivity all important for Thailand

Taking Stock

- Started documenting structural change patterns in the data
  - Economies undergo deep transformations along growth process
  - Similar patterns across countries, different points in time
  - But not identical (importance of trade, technology available, etc.)
- Developed a benchmark multi-sector extension if Ramsey NGM.
- Used it to study supply-side drivers of SC
- Discussed Stringent constraints for BGP and saw STrAP concept to relax it.