Structural Transformation: Demand-Side Theories

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Two Views on Drivers of Structural Change

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Supply

Demand

Two Views on Drivers of Structural Change

Trends in Productivity, K-shares

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Trends in Productivity, K-shares

Non-homothetic Engel Curves

Two Views on Drivers of Structural Change

Trends in Productivity, K-shares	Non-homothetic Engel Curves

Introduction

- We will maintain our benchmark model, but focus on the role of consumption aggregator.
- Capture nonhomotheticities $\rightarrow$ Engel Curves.
- **All** supply-theories we saw **cannot** account for the co-movement of nominal and real consumption along the consumption path under the gross-complementarity assumption.
    - Counterfactual prediction
    - Nonhomothetic demand can account for this positive correlation

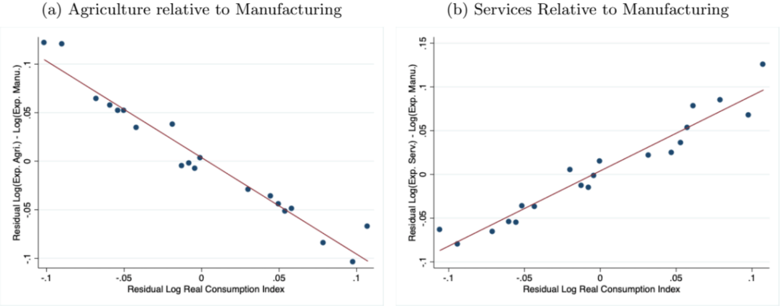
Engel's Curve: Prima facie Evidence for Nonhomotheticity

- One of the first empirical findings in economics documented.
- Originally documented by Ernst Engel (18211896), relationship between goods expenditure and income.
- **Engel's law** as income grows, spending on food becomes a smaller share of income.
- Validated across households and countries, and for more spending categories.

Engel's Curve across Households: US CEX

Plot: Partial correlation of rel. exp. shares on total expenditure

$$\log\left(\frac{\text{share}_i}{\text{share}_j}\right) = \alpha \log p_i + \beta \log p_j + \gamma \log \text{Expenditure} + \text{hh controls}$$

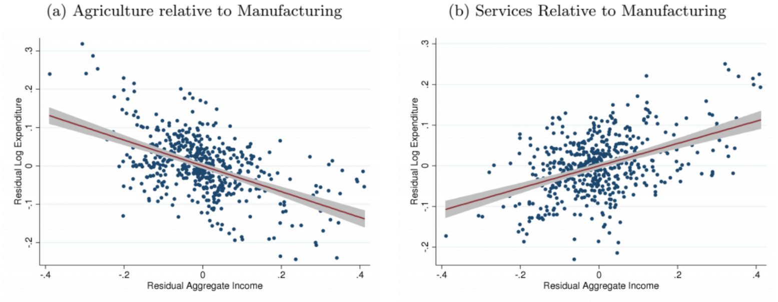


Notes: These plots depict the (binned) residuals corresponding to the average value of 20 equal-sized bins of the data. The red line depicts the linear regression between the residualized variables.

Engel's Curve across OECD countries

Plot: Partial correlation of rel. exp. shares on total income pc

$$\log\left(\frac{\text{share}_i}{\text{share}_j}\right) = \alpha \log p_i + \beta \log p_j + \gamma \log \text{Expenditure} + \text{country FE}$$



Notes: Data for OECD countries, 1970-2005. Each point corresponds to a country-year observation after partialling-out sectoral prices and country fixed effects. The red line depicts the OLS fit, the shaded regions, the 95% confidence interval.

Back to Our Benchmark Model of Structural Change

- Consider an economy consisting of three sectors:
  - Agriculture (a), manufacturing (m) and services (s).
- Assume a representative consumer and a closed economy.
  - Assume inelastic labor supply
  - Representative agent rules out inequality.
- Output of three sectors used to create two aggregates:
  1. consumption C,
  2. investment X.
- Production in each sector uses capital and labor, although potentially in different proportions.

Representative Agent Problem

- Rep. Agent maximizes:

$$\max_{C(t),X(t),K(t),B(t)} \int_{t=\tau}^{\infty} e^{-\rho(t-\tau)} U(C(C_a(t), C_m(t), C_s(t)))dt,$$

s.t.

$$P_c(t)\,C(t) + P_x(t)\,X(t) + P_c(t)\,\dot{B}(t) =$$
$$W(t)\,L + R(t)\,K(t) + r(t)\,P_c(t)\,B(t),$$

and

$$\dot{K}(t) = X(t) - \delta K(t).$$

- All action from demand-side theory comes from $U(C(\cdot))$
    - $U(\cdot)$ affects intra-temporal problem only,
    - $C(\cdot)$ affects both.

Classic Example: Rebelo, Kongsamut and Xie (2001)

- Let $U(\cdot) = \log(\cdot)$
- Specify $C(\cdot)$ to be a generalized Stone-Geary:

$$C_t = \left(\omega_a^{\frac{1}{\varepsilon}} (C_{at} - \bar{C}_a)^{\frac{\varepsilon-1}{\varepsilon}} + \omega_m^{\frac{1}{\varepsilon}} (C_{mt})^{\frac{\varepsilon-1}{\varepsilon}} \omega_s^{\frac{1}{\varepsilon}} (C_{st} + \bar{C}_s)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

  with $\bar{c}_a, \bar{c}_s > 0$.
- **Assume** income and prices such that can achieve at least $\bar{C}_a$.
  - Many (most?) nonhomothetic preferences need to impose restriction on income and prices.

RKX (2001): Supply Side

- Idea: try to keep it as innocuous as possible.
- Sectoral production: Cobb-Douglas with identical shares, α,

$$F_i = A_i(L_i)^{1-\alpha}K_i^{\alpha}$$

- Investment: done with manufacturing.
  - RKX assume it is separately done with specific capital and labor, does not matter qualitatively.
- We showed α's$\rightarrow$ capital-labor equalized across sectors.
- Normalize manuf. price to 1,

$$\frac{P_i}{P_m} = P_i = \frac{A_m}{A_i}$$

RKX: Supply-Side Aggregation Property

- Combining the spending on different sectors,

$$Y_t = P_{at} C_{at} + C_{mt} + P_{st} C_{st} + X_t = K_t^{\alpha} A_{mt}$$

- The model aggregates in the production side
- Equivalent to have one production function that produces a single good that can be turned into either consumption or investment, according to linear technology that has a price in it.

RKX: Consumption Decision

- Divide problem in 2 sub-problems of how to divide
  1. income between total consumption C_t and savings (Inter-temporal)
  2. income spent in consumption of different goods (Intra-temporal prob).
- By assumption income is high enough to consume three types of goods,

$$\frac{1}{C_t}\omega_a^{\frac{1}{\varepsilon}}(C_{at}-\bar{C}_a)^{-\frac{1}{\varepsilon}}C_t^{\frac{1}{\varepsilon}} = \lambda_t P_{at} \quad (1)$$

$$\frac{1}{C_t}\omega_m^{\frac{1}{\varepsilon}}(C_{mt})^{-\frac{1}{\varepsilon}}C_t^{\frac{1}{\varepsilon}} = \lambda_t P_{mt} \quad (2)$$

$$\frac{1}{C_t}\omega_s^{\frac{1}{\varepsilon}}(C_{st}+\bar{C}_s)^{-\frac{1}{\varepsilon}}C_t^{\frac{1}{\varepsilon}} = \lambda_t P_{st} \quad (3)$$

where λ_t is the Lagrange multiplier on the BC.

RKX: Consumption Decision ct'd

- Manipulating we have that

$$\frac{1}{C_t} = \lambda_t \underbrace{\left(\omega_a (P_{at})^{1-\varepsilon} + \omega_m (P_{mt})^{1-\varepsilon} + \omega_s (P_{st})^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}}_{\equiv P_t} \quad (4)$$

- Adding the FOCs we have that

$$P_{at} C_{at} + P_{mt} C_{mt} + P_{st} C_{st} = P_t C_t + P_{at} \bar{C}_a - P_{st} \bar{C}_s \quad (5)$$

- Role of non-homotheticity: introduce a "time varying endowment" $-P_{at} \bar{C}_a + P_{st} \bar{C}_s$ to an otherwise standard economy.

RKX: Equilibrium relationship for Structural Change

- Using the ratios of FOCs we have evolution relative shares:

$$\left(\frac{P_{at}}{P_{mt}}\right)^{\varepsilon} \frac{C_{at} - \bar{C}_a}{C_{mt}} = \frac{\omega_a}{\omega_m} \quad (6)$$

$$\left(\frac{P_{st}}{P_{mt}}\right)^{\varepsilon} \frac{C_{at} + \bar{C}_s}{C_{mt}} = \frac{\omega_s}{\omega_m} \quad (7)$$

- Use ratio of expenditure on composite consumption and expenditure on manufacturing (info on level of manu. sh.)

$$\frac{P_t C_t}{P_{mt} C_{mt}} = \left[\frac{\omega_a}{\omega_m}\left(\frac{A_{mt}}{A_{at}}\right)^{(1-\varepsilon)} + 1 + \frac{\omega_s}{\omega_m}\left(\frac{A_{mt}}{A_{st}}\right)^{(1-\varepsilon)}\right] \quad (8)$$

RKX (2001): Structural Change with BGP

- Assume productivity growth is constant and identical across sectors
$$\frac{\dot{A}_i}{A_i} = \gamma \qquad \text{for all } i \in \{a, m, s\}$$
- This implies that relative prices are constant over time!
- Assume $\varepsilon = 1$, Cobb-Douglas.
  - These are called Stone-Geary preferences
- Euler equation (exercise: check it is correct)
$$\frac{\dot{E}}{E} = R - \delta - \rho \tag{9}$$
where $E = P_t C_t$.
- BGP requires R and $\frac{\dot{E}}{E}$ constant.

KRX (2001): Structural Change with BGP ct'd

- Looking at aggregate budget constraint,

$$P_t C_t + P_{at} \bar{C}_a - P_{st} \bar{C}_s = L^{1-\alpha} K_t^{\alpha} A_{mt} + \dot{K} - \delta K_t. \qquad (10)$$

- Constant $R = \alpha K^{\alpha-1} A_m$ implies K grows at $\gamma/(1-\alpha)$.
- RHS of BC grows at $\gamma/(1-\alpha) \Rightarrow$ LHS grows at same rate.
- If $P_{at} \bar{C}_a - P_{st} \bar{C}_s \neq 0$, then it must grow at rate $\gamma/(1-\alpha)\ldots$
  - but prices do not grow! Not BGP in general!
  - If we had separate investment sector as original KRX with own TFP, this contradiction is qualified but still goes through.
- Since in general these are different, need to impose that $P_{at} \bar{C}_a - P_{st} \bar{C}_s = 0$, implying

$$\frac{\bar{C}_a}{\bar{C}_s} = \frac{A_{a0}}{A_{s0}} \qquad (11)$$

KRX (2001): Behaviour Along BGP

- Imposing 11 ensures a BGP (aka Generalized BGP).
- Consumption along GBGP satisfies

$$
\begin{aligned}
C_{at} &= \omega_a \frac{P_t C_t}{P_{at}} + \bar{C}_a && (12)\\
C_{mt} &= \omega_m \frac{P_t C_t}{P_{mt}} && (13)\\
C_{st} &= \omega_s \frac{P_t C_t}{P_{st}} - \bar{C}_s && (14)
\end{aligned}
$$

- Assumption of constant growth implies that

$$
\frac{P_{it}}{P_t} = \frac{P_{i0}}{P_0} \qquad i \in \{a, m, s\} \tag{15}
$$

KRX (2001): Behaviour Along GBGP

- C_{at} grows at a slower rate than C_t, C_m at the same, and C_s faster
- As relative prices are constant, the same is true for $P_{it}C_{it}/P_tC_t$.
- As total consumption expenditures are constant share of total output, the same properties carry over to L_{it} and $P_{it}C_{it}/Y_t$.

KRX (2001): Merits/Limitations

- Can account for rise of services, decline of agriculture.
- Relies on a knife-edge case, (11). Ngai Pisarides say

  > *KRX (2001) obtain their results by imposing a restriction that maps some of the parameters of their Stone-Geary utility function onto the parameters of the production functions, abandoning one of the most useful conventions of modern macroeconomics, the complete independence of preferences and technologies.*

- Will show that it is possible to reproduce KRX (2001) without knife-edge Eq. (11) using alternative demand system.

KRX (2001): Merits/Limitations ct'd

- Cannot deliver hump-shape for manufacturing by construction.
- Relative prices are constant (counterfactual)
  - Real and nominal variables exhibit same behaviour.
- Taken seriously, in poor economies the model implies zero consumption of services, which is not true.
- Demand becomes asymptotically homothetic: at odds with empirical evidence.

Boppart (2015)

- Incorporate income effects and price effects in a tractable fashion using PIGL preferences
- Key point (made in Buera Kaboski 2009, JEEA): relative price of goods to services has declined at a lower rate than relative expenditure share of goods.
  - Cannot replicate this with CES demand even if Leontief.
- Model is consistent with Kaldor Facts plus cross-sectional expenditure structure differences: poor people spend more on food than rich people.
  - Departure from representative agent, but quasi-aggregation.

Poor HH Spend Larger fraction of Budget on Goods

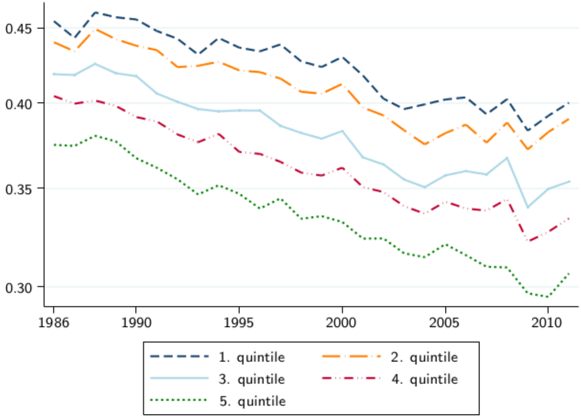


Figure 5 : **Cross-sectional variation in the expenditure shares of goods**

Boppart (2015): Set-up

- Mass 1 of HH, $i \in [0, 1]$
- HH i endowed with l_i units of labor and a_i of wealth
- Use PIGL Preferences (more on next slide),

$$U_i(0) = \int_0^\infty e^{-\rho t} V(P_G(t), P_S(t), e_i(t)) dt$$

- $V(P_G(t), P_S(t), e_i(t))$ is an instantaneous **indirect utility** function, where $P_G(t)$, $P_S(t)$ are prices of goods and services and e_i is the nominal expenditure of HH i
  - Dual of PIGL does not have closed form
  - Still prefs. are well-defined (some restrictions as in G. Stone-Geary)
  - Widely used in empirical demand estimation.
- Note $V(\cdot)$ plays both $U(\cdot)$ and $C(\cdot)$ role (no separate margins)

Boppart (2015): Preferences

- Case of "Price Independent Generalized Linear" Pref:

$$V(P_G(t), P_S(t), e_i(t)) = \frac{1}{\varepsilon}\left(\frac{e_i(t)}{P_S(t)}\right)^{\varepsilon} - \frac{\nu}{\gamma}\left(\frac{P_G(t)}{P_S(t)}\right)^{\gamma} - \frac{1}{\varepsilon} + \frac{\nu}{\gamma}$$

  with $\nu, \gamma \geq 0$ and $0 \leq \varepsilon \leq \gamma < 1$.

- Individual Expenditure System

$$\eta_G^i(t) \equiv \frac{P_G(t)x_G^i(t)}{e_i(t)} = \nu e_i^{-\varepsilon} P_S(t)^{\varepsilon-\gamma} P_G(t)^{\gamma}$$

  as $e_i \to \infty$, $\eta_G^i = 0$, also $\eta_S = 1 - \eta_G$.

- Aggregate Expenditure System (X, E denote aggregates)

$$\eta_G(t) \equiv \frac{P_G(t)X_G(t)}{E(t)} = \nu E^{-\varepsilon} P_S(t)^{\varepsilon-\gamma} P_G(t)^{\gamma} \phi(t)$$

  $\phi(t) \equiv \int_0^1 \left(\frac{e_i(t)}{E(t)}\right)^{1-\varepsilon} di$: scale invariant measure of inequality.

Boppart (15): Nonhomotheticity, $\varepsilon > 0$, defined for $e_i > \bar{e}$

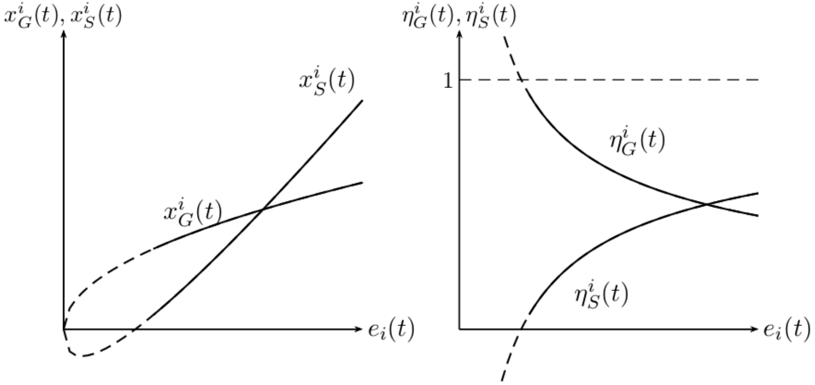


Figure 8 : Engel curves

Figure 9 : Expenditure shares

- "Sufficiently Rich" condition: $e_i^{\epsilon} > \left(\frac{1-\epsilon}{1-\gamma}\right) \nu P_G^{\gamma} P_S^{\epsilon-\gamma}$
- Qualitatively similar to Stone-Geary. (How to get $\eta_i^G = 1$?)

Boppart (2015): Derivations of Demand Side

- Apply Roy's identity (implicit function theorem) to derive Marshallian demands,

$$x_i^G(P_G, P_S, e_i) = -\frac{\frac{\partial V}{\partial P_G}}{\frac{\partial V}{\partial e_i}}$$

  and analogously for S.
- Intertemporal Problem solved as usual, using e_i directly. E.g., Hamiltonian is

$$\mathcal{H} = V(P_G, P_S, e_i) + \lambda \left(a_i r + W l_i - e_i\right)$$

Boppart (2015): Dynamics

- Euler Equation

$$(1-\varepsilon)g_{e_i}(t)+\varepsilon g_{P_s}(t)=r(t)-\rho \tag{16}$$

where g_{e_i} is the growth rate of expenditures and g_{P_s} growth rate of rel. price of services.

- All individual expenditures grow at same rate $\rightarrow \phi(t)=\phi$.

Boppart (2015): Supply Side

- Production of G, S is CD with exogenous Hicks-neutral productivity growth.
  - Relative price between the two depends on relative productivities.
  - Assume sectoral prod. growth is constant (and possibly different).
- Investment: AK technology. Investment produced only capital, $A > \delta$.
  - Normalize price investment to 1.
  - "Cheat" to have constant interest rate (done in a few papers).

Boppart (2015): GBGP

- Constant interest rate, savings rate and income share.
- Expenditure, wages, aggregate capital grow at constant rates.
- The price of consumption goods rel. to services changes at the exogenous tech. progress.
- Expenditure share to goods decreases at a constant rate
- Capital and labor allocated to goods sector grow at constant rates

Boppart (2015): Merits and Limitations

- Delivers structural change with changes in relative prices and income effects.
- Fits stylized facts.
- Cross sectional implications that match well data.
- EIS is linked to nonhomotheticity term ε.
- Only 2 sectors can be non-homothetic, manufacturing vs. agriculture?
    - Alder, Boppart, Mueller (2022, AEJ Macro): extend more than two sectors, while preserving aggregation properties.

Buera Kaboski (2012)

- Analyze the role of home-production vs market purchased services.
- Continuum of goods and services.
- Each good is an input for a service.
- Labor is used for producing goods, goods and labor used to produce services.
- Consumer want to consume at most 1 unit of each service.
- Trade-off: market production of services is more efficient but home production generates more utility (eg, private car vs. bus)
- As an economy develops the marginal services that are added feature higher benefits to market than home production.

Buera Kaboski (2012)

- The combination of technological change plus the changing nature of the marginal services being brought into the economy can introduce interesting dynamics for how activity shifts between the market and home sectors.
- If production shifts toward the market and away from the home, this will be recorded as an increase in the size of the market service sector relative to the goods sector.
- When thinking about growth and structural transformation it is important to think about the new goods and services that are associated with growth
- They argue that rising return to skill is intimately connected to the structural transformation of economic activity towards services.

Modelling Nonhomotheticities through Implicit Additivity

Pigou's Law (Deaton, 1974)

Direct additive preferences imply that own price elasticities are proportional to income elasticities.

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- Consider additive preferences: $C = F\left(\sum_k f_k\left(C_k\right)\right)$.

Modelling Nonhomotheticities through Implicit Additivity

Pigou's Law (Deaton, 1974)

Direct additive preferences imply that own price elasticities are proportional to income elasticities.

- Consider additive preferences: $C = F\left(\sum_k f_k\left(C_k\right)\right).$
- For $\{P_k\}$ prices, E expenditure, $\{s_k\}$ expenditure shares:

$$\frac{\text{Price Elasticity}_k}{\text{Income Elasticity}_k} = \frac{\eta_{C_k}^{P_k}}{\eta_{C_k}^{E}} = -A - s_k\left(1 - A\,\eta_{C_k}^{E}\right) =_{s_k \to 0} -A$$

with $A \equiv \sum_k s_k \left(\eta_{f_k'}\right)^{-1}$.

- Analogous result with indirect utility (then for difference).

Motivation

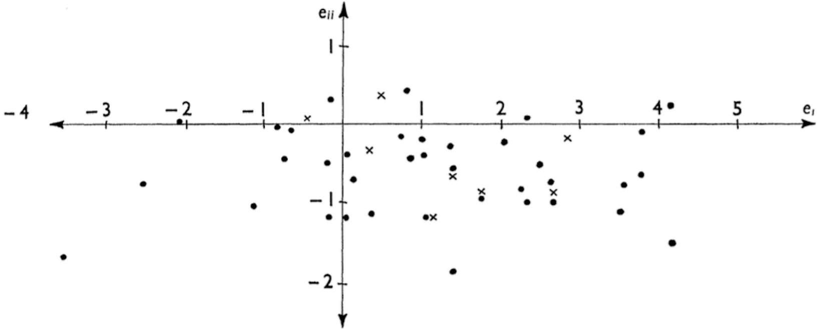

FIG. 1. Income and price elasticities for double-log model.

Motivation

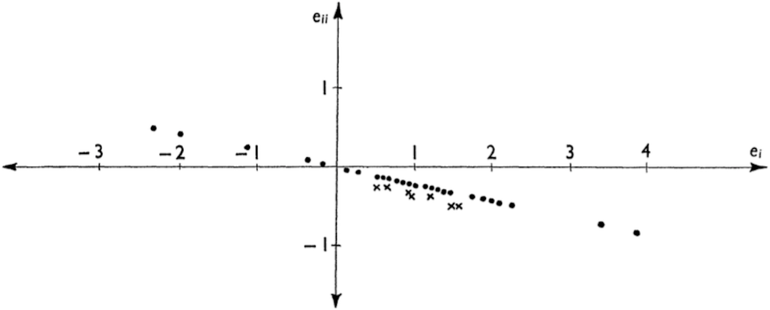

FIG. 2. Income and price elasticities for a directly additive model.

Motivation

Deaton (1974) concludes:

> *"...the assumption of additive preferences is almost certain to be invalid in practice and the use of demand models based on such an assumption will lead to severe distortion of measurement. So that if the price to be paid for the theoretical consistency of demand models is the necessity of assuming additive preferences, then the price is too high."*

Implicit Additivity is Needed to Preserve CES

Gorman (1965)

Additively Separable Utility & Constant Own Price ES
$\iff$ Homothetic CES.

- Demand systems that are explicitly additive have nonconstant own ES if they are not CES (e.g., Stone Geary, PIGL)
- If want to present CES need to depart from them
- Even if ex-post want to impose particular relationship between elasticities of substitutions, need more flexible framework to test for it.

Comin, Lashkari and Mestieri (2021)

- Consider baseline model, with Ngai-Pissarides assumptions
  - CRRA intertemporal pref. w/ parameter θ
  - Cobb-Douglas w/same α's,
  - Investment w/ Manuf. only.
- Agents' intratemporal utility is implicitly defined through

$$\sum_{i=1}^{I} \left(\frac{C_i(t)}{C^{\epsilon_i}(t)} \right)^{\frac{\sigma-1}{\sigma}} = 1, \tag{17}$$

  - $\epsilon_i > 0$, parametrizes income elasticity of sector i,
  - σ is the elasticity of substitution which is constant.
  - Prefs. called nonhomothetic CES
  - They assume complements $\sigma \in (0,1)$ but $\sigma > 1$ is also admissible, and $\min_i\{\epsilon_i\} > 1 - \theta$ to ensure strict concavity of Hamiltonian.
- Derive demand through cost minimization or utility maximization.

Deriving NhCES through Cost Minimization

- According to Berthold Herrendorf, this is the most intuitive derivation.
- Begin deriving through minimizing cost of obtaining C

$$\min_{\{C_i\}} \sum_{i=1}^{I} P_i C_i \quad \text{subject to} \quad \sum_{i=1}^{I} \left(\frac{C_i(t)}{C^{\epsilon_i}(t)} \right)^{\frac{\sigma-1}{\sigma}} = 1$$

- The Lagrangian is in this case:

$$\mathcal{L} = \sum_{i=1}^{I} P_i C_i + \lambda \left(\sum_{i=1}^{I} \left(\frac{C_i(t)}{C^{\epsilon_i}(t)} \right)^{\frac{\sigma-1}{\sigma}} - 1 \right)$$

- The FOC with respect to C_i is

$$P_i C_i = \lambda \frac{\sigma-1}{\sigma} \left(\frac{C_i(t)}{C^{\epsilon_i}(t)} \right)^{\frac{\sigma-1}{\sigma}}$$

Deriving NhCES through Cost Minimization

- Denote total expenditure by E. Sum across all i to obtain:

$$E = \sum_i P_i C_i = \lambda \frac{\sigma - 1}{\sigma} \underbrace{\sum_i \left(\frac{C_i(t)}{C^{\epsilon_i}(t)} \right)^{\frac{\sigma-1}{\sigma}}}_{=1} = \lambda \frac{\sigma - 1}{\sigma}$$

- Use this result in the FOC to obtain expenditure share of i

$$\frac{P_i C_i}{E} = \left(\frac{C_i(t)}{C^{\epsilon_i}(t)} \right)^{\frac{\sigma-1}{\sigma}}$$

or alternatively

$$\frac{P_i C_i}{E} = \left(\frac{P_i C^{\epsilon_i}}{E} \right)^{1-\sigma}$$

- Rearranging, we obtain the demand for C_i

$$C_i = \left(\frac{P_i}{E} \right)^{-\sigma} C^{\epsilon_i(1-\sigma)}$$

Deriving Demand of NhCES through Cost Minimization

- Finally, we can define E in terms of optimal choices:
- Combining the expression for C_i and definition of E

$$E = \sum_i P_i C_i = \sum_i P_i \left(\frac{P_i}{E}\right)^{-\sigma} C^{\epsilon_i(1-\sigma)}$$

rearranging, we obtain the expenditure function

$$E = \left[\sum_i \left(C^{\epsilon_i} P_i\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

- Next show we obtain the same by utility maximization.

Deriving Demand of NhCES through Utility Maximization

- The utility maximization problem is

$$\max_{\{C_i\}} \quad C \quad \text{subject to} \quad \sum_{i=1}^{I} P_i C_i = E, \text{ and } \sum_{i=1}^{I} \left(\frac{C_i(t)}{C^{\epsilon_i}(t)} \right)^{\frac{\sigma-1}{\sigma}} = 1$$

- Lagrangian is simply:

$$\mathcal{L} = C + \rho \left(\sum_{i=1}^{I} \left(\frac{C_i(t)}{C^{\epsilon_i}(t)} \right)^{\frac{\sigma-1}{\sigma}} - 1 \right) + \lambda \left(E - \sum_i P_i C_i \right)$$

- FOCs involve partial derivatives only, thus partial wrt C_i is[1]

$$\rho \frac{\sigma - 1}{\sigma C_i} \left(\frac{C_i}{C^{\epsilon_i}} \right)^{\frac{\sigma-1}{\sigma}} = \lambda P_i$$

[1] Think how you would apply this procedure if C was Cobb Douglas to convince yourself about the partial derivatives.

Deriving Demand through Utility Maximization II

- Rearrange and sum over all i to obtain:

$$\rho \frac{\sigma - 1}{\sigma} \underbrace{\sum_i \left(\frac{C_i}{C^{\epsilon_i}} \right)^{\frac{\sigma-1}{\sigma}}}_{=1} = \lambda \sum_i P_i C_i = \lambda E \Rightarrow \frac{\rho}{\lambda} \frac{\sigma - 1}{\sigma} = E$$

- Plug back into the FOC we find that expenditure shares are

$$\frac{P_i C_i}{E} = \left(\frac{C_i}{C^{\epsilon_i}} \right)^{\frac{\sigma-1}{\sigma}}$$

- Demand for good i is thus

$$C_i = \left(\frac{P_i}{E} \right)^{-\sigma} C^{\epsilon_i(1-\sigma)}$$

- We thus obtain the same demands.

NhCES Properties: Expenditure Elasticity

- The Expenditure Elasticity is

$$\frac{\partial \ln(P_i C_i)}{\partial \ln E} = \frac{\partial \ln C_i}{\partial \ln E} = \frac{\partial \left(\sigma(\ln E - \ln P_i) + \epsilon_i(1-\sigma)\ln C\right)}{\partial \ln E}$$
$$= \sigma + \epsilon_i(1-\sigma)\frac{\partial \ln C}{\partial \ln E} = \sigma + \epsilon_i(1-\sigma)\frac{1}{\frac{\partial \ln E}{\partial \ln C}}$$

- Compute last derivative from expenditure func

$$\frac{\partial \ln E}{\partial \ln C} = \frac{\sum_i \epsilon_i \left(C^{\epsilon_i} P_i\right)^{1-\sigma}}{\sum_i \left(C^{\epsilon_i} P_i\right)^{1-\sigma}} = \sum_i \epsilon_i \left(\frac{C^{\epsilon_i} P_i}{E}\right)^{1-\sigma} = \sum_i \epsilon_i \frac{P_i C_i}{E}$$

- Defining $\bar{\epsilon} \equiv \sum_i \epsilon_i \frac{P_i C_i}{E}$ and substituting in the first equation,

$$\frac{\partial \ln(P_i C_i)}{\partial \ln E} = \sigma + (1-\sigma)\frac{\epsilon_i}{\bar{\epsilon}}$$

- Goods can start as luxuries and become necessities.

Euler Equation Derivation

- Set up the Lagrangian assuming some labor income W_t:

$$\max_{A_t, C_t} \quad \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\theta}}{1-\theta} + \sum_{t=0}^{\infty} \lambda_t \left(W_t + R_t A_t - A_{t+1} - E(C_t) \right)$$

where $E(C_t)$ is the expenditure function already derived.

- Take FOC wrt C_t and A_t are

$$\lambda_t = R_{t+1} \lambda_{t+1}$$
$$\beta^t C_t^{-\theta} = \lambda_t \frac{\partial E_t}{\partial C_t} = \bar{\epsilon}_t \lambda_t \frac{E_t}{C_t}$$

where in the last line I use the result for $\partial \ln E / \partial \ln C$.

Derivation of the Euler Equation ct'd

- Combine these two first-order conditions, define $P_t = E_t / C_t$

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\theta} = \frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t} \frac{P_{t+1}}{P_t} \frac{\lambda_{t+1}}{\lambda_t} = \frac{\bar{\epsilon}_{t+1}}{\bar{\epsilon}_t} \frac{P_{t+1}}{P_t} \frac{1}{R_{t+1}}$$

- We have standard substitution effect through P_{t+1}/P_t
- Additional effect from nonhomotheticity, $\bar{\epsilon}_{t+1}/\bar{\epsilon}_t$
  - Generates a nonconstant EIS.

Continuous Time Rendition of HH Decisions

- Budget constraint (labor and capital income spent or saved):

$$\dot{\mathcal{A}} + E\,(t) \leq W(t) + r\,(t)\,\mathcal{A}\,(t) \tag{18}$$

HH Optimal Choices Given $[P\,(\cdot)\,, r\,(\cdot)\,, W(t)(\equiv 1)]_{t=0}^{\infty}\,, \mathcal{A}(0)$

Notation: $\bar{p} = \sum_{i=1}^{I} \Omega_i p_i$, $p_i = \ln P_i$ *except* for $r(t)$.

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Notation: $\bar{p} = \sum_{i=1}^{I} \Omega_i p_i$, $p_i = \ln P_i$ *except* for $r(t)$.
Optimal paths $C(t), \{C_i(t)\}$

$$\dot{c} \equiv \frac{\dot{C}(t)}{C(t)} = \frac{r(t)-\rho-\dot{\bar{p}}(t)\left[1+(1-\sigma)Cov\left(\frac{\epsilon_i}{\bar{\epsilon}(t)}, \frac{\dot{p}_i(t)}{\dot{\bar{p}}(t)}; t\right)\right]}{\vartheta+\bar{\epsilon}(t)\left[1+(1-\sigma)Var\left(\frac{\epsilon_i}{\bar{\epsilon}(t)}; t\right)\right]-1}, \tag{19}$$

$$\Omega_i(t) \equiv \frac{P_i(t)\,C_i(t)}{E(t)} = \left(\frac{P_i(t)}{E(t)} C(t)^{\epsilon_i}\right)^{1-\sigma} \quad \forall i, \tag{20}$$

$$E(t) = \left(\sum_{i=}^{I} \left(P_i(t)\,C(t)^{\epsilon_i}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \tag{21}$$

plus transversality condition $\lim_{t\to\infty} e^{-(\rho-\eta)t} \frac{\mathcal{A}(t)}{E(t)} C(t)^{1-\vartheta} \frac{1}{\bar{\epsilon}(t)} = 0$.

Discussion of Properties the Household Behavior

- Deviations Euler equation from homothetic case:
  - Term $\overline{\epsilon}(t)\left[1+(1-\sigma)\,Var\left(\frac{\epsilon_i}{\overline{\epsilon}(t)};t\right)\right]-1$ implies that the concavity of C (IES) depend on t: $p_i(t)$, $C(t)$.
  - Term $(1-\sigma)\,Cov\left(\frac{\epsilon_i}{\overline{\epsilon}(t)},\frac{\dot{p}_i(t)}{\dot{\overline{p}}(t)};t\right)$ consumption grows faster if prices fall faster for more income-elastic goods.
- Growth rates of c and e satisfy: [divisa index and line integral]

$$\overline{\epsilon_i}(t)\,\dot{c}(t)=\dot{e}(t)-\overline{\dot{p}_i}(t)$$

- Expenditure shares in sector i grow according to

$$\begin{aligned}\dot{\omega}_i(t)&=(1-\sigma)\left(\epsilon_i\dot{c}(t)+\dot{p}_i(t)-\dot{e}(t)\right),\\&=(1-\sigma)\left[\left(\epsilon_i-\overline{\epsilon}(t)\right)\dot{c}(t)+\dot{p}_i(t)-\overline{\dot{p}}(t)\right],\end{aligned}\tag{22}$$

income and price effects at work.

Comin, Lashkari and Mestieri (2021)

- Build on the observation that income elasticities do not seem to decrease as income goes up (as predicted by Stone Geary)
  - Log-linear specification of Engel curves provides a good description, Aguiar and Bils for US (AER 2016), Young (2013 QJE, JPE 2012) (includes many developing countries)
  - CLM document similar real income elasticities of agriculture, manufacturing and services in US and India using consumption expenditure surveys.
  - **Key Insight**: Engel Curves do not level-off as income grows
- Use nonhomothetic CES demand:
  - Nonvanishing nonhomotheticity.
  - Accommodates an arbitrary number of sectors.
- Show that NhCES provides a parsimonious fit of the data.

Shortcoming of Stone-Geary Preferences

$$C_t(C_{at}, C_{mt}, C_{st}) \;=\; \left((C_{at} - \overline{c}_a)^{\frac{\sigma-1}{\sigma}} + C_{mt}^{\frac{\sigma-1}{\sigma}} + (C_{st} + \overline{c}_s)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

- Asymptotically Homothetic (non-homotheticity is transitional)

$$C_{it} \gg \overline{c}_i \quad \Longrightarrow \quad \varepsilon_i \equiv \frac{\partial \ln C_{it}}{\partial \ln C_t} \to 1.$$

- Quantitatively, bad fit for different income levels.

Estimation: Log-Linear Demand System

- Within period demand

$$C_{it} = \zeta_i \left(\frac{p_{it}}{P_t}\right)^{-\sigma} \left(\frac{E_t}{P_t}\right)^{\varepsilon_i},$$

- Taking ratio of demand i and j (and use market clearing):

$$\begin{aligned}
\log\left(\frac{C_{it}}{C_{jt}}\right) &= \alpha_{ij} - \sigma \log\left(\frac{p_{it}}{p_{jt}}\right) + (\epsilon_i - \epsilon_j)\log C_t, \\
\log\left(\frac{\omega_{it}}{\omega_{jt}}\right) &= \alpha_{ij} + (1-\sigma) \log\left(\frac{p_{it}}{p_{jt}}\right) + (\epsilon_i - \epsilon_j)\log C_t, \\
\log\left(\frac{L_{it}}{L_{jt}}\right) &= \alpha_{ij} + (1-\sigma) \log\left(\frac{p_{it}}{p_{jt}}\right) + (\epsilon_i - \epsilon_j)\log C_t.
\end{aligned}$$

where ω_{it} denotes expenditure share in sector i at time t.

Empirical Application: Country Panel

- GGDC 10-Sector Database for sectoral data, Barro-Ursua for real consumption.
- 9 countries Asia, 9 in Europe, 9 in Latin America, US and South Africa. Period: 1947-2005.

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$$\log\left(\frac{L^c_{a,t}}{L^c_{m,t}}\right) = \alpha^c_{am} + (1-\sigma)\log\left(\frac{p^c_{a,t}}{p^c_{m,t}}\right) + (\varepsilon_a - \varepsilon_m)\log C^c_t + \nu^c_{am,t},$$
$$\log\left(\frac{L^c_{s,t}}{L^c_{m,t}}\right) = \alpha^c_{sm} + (1-\sigma)\log\left(\frac{p^c_{s,t}}{p^c_{m,t}}\right) + (\varepsilon_s - \varepsilon_m)\log C^c_t + \nu^c_{sm,t}.$$

- Control for sectoral exports and imports,

$$\log\left(\frac{X^c_{i,t}}{M^c_{i,t}}\right).$$

Baseline Econometric Specification: Derivation

- Write price index in terms of observables.

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- Substitute in $i \neq m$,

$$\log \left(\frac{\omega_{it}^n}{\omega_{mt}^n} \right) = (1-\sigma) \log \left(\frac{p_{it}^n}{p_{mt}^n} \right) + (1-\sigma) \left(\frac{\epsilon_i}{\epsilon_m} - 1 \right) \log \left(\frac{E_t^n}{p_{mt}^n} \right) + \left(\frac{\epsilon_i}{\epsilon_m} - 1 \right) \log \omega_{mt}^n + \zeta_i^n.$$

Baseline Econometric Specification

- Estimating equation is

$$\log\left(\frac{\omega_{it}^n}{\omega_{mt}^n}\right) = A_1 \log\left(\frac{p_{it}^n}{p_{mt}^n}\right) + A_2 \log\left(\frac{E_t^n}{p_{mt}^n}\right) + A_3 \log \omega_{mt}^n + \zeta_i^n + \nu_{it}^n,$$

  with the constraint that

$$A_1 A_3 = A_2.$$

  - Makes clear that only relative ϵ_i/ϵ_m are identified.
  - Alternative normalizations are possible, e.g., $\sum_i \epsilon_i = 1$. Details

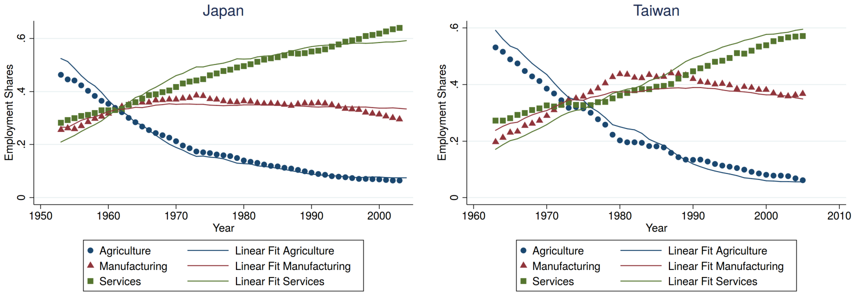
Baseline Estimation

Dep. Var.:	World		
Rel. Emp.	(1)	(2)	(3)
σ	0.66	0.75	0.72
	(0.19)	(0.11)	(0.11)
$\varepsilon_a - \varepsilon_m$	-0.81	-1.09	-1.03
	(0.24)	(0.10)	(0.14)
$\varepsilon_s - \varepsilon_m$	0.32	0.32	0.32
	(0.08)	(0.10)	(0.13)
Obs.	1006	1006	916
$c \cdot sm$ FE	N	Y	Y
Trade Controls	N	N	Y

Note: Std. Errors Clustered by Country

Asia

Uses World Estimates for All Elasticities, $\{\sigma, \varepsilon_a - \varepsilon_m, \varepsilon_s - \varepsilon_m\}$



Compare with Stone-Geary Estimation Results

$$\left(\zeta_{am}^{c}(C_{at}-\overline{c}_{a})^{\frac{\sigma-1}{\sigma}}+C_{mt}^{\frac{\sigma-1}{\sigma}}+\zeta_{sm}^{c}(C_{st}+\overline{c}_{s})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

- Estimate same model with Stone-Geary within period utility
- **Same** number of parameters to estimate

$$\{\sigma,\bar{c}_a,\bar{c}_s,\zeta_{am}^{c},\zeta_{sm}^{c}\}$$

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- Estimate same model with Stone-Geary within period utility
- **Same** number of parameters to estimate

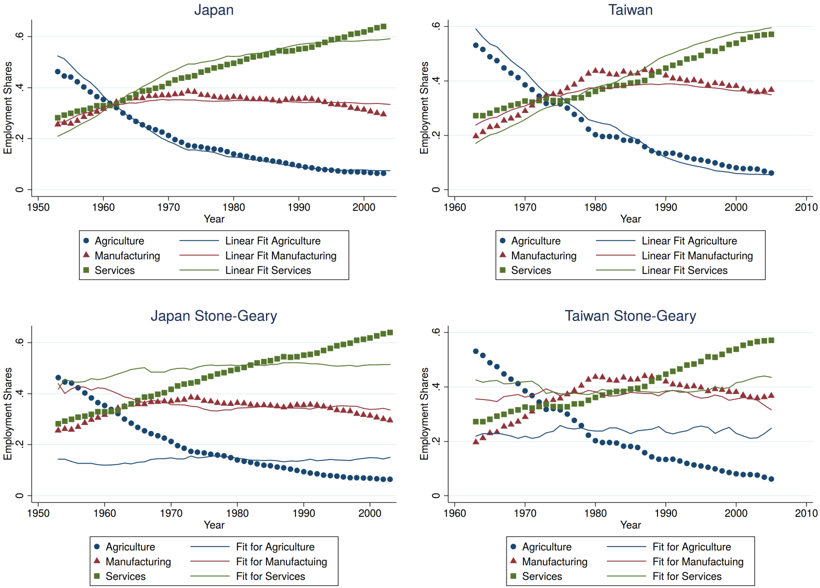
$$\{\sigma, \bar{c}_a, \bar{c}_s, \zeta_{am}^c, \zeta_{sm}^c\}$$

- Imposes correlation between income and price elasticities

$$\sigma_{ij} = \sigma \varepsilon_i \varepsilon_j$$

Asia - Nonhomothetic CES vs. Stone-Geary

Uses World Estimates for All Elasticities, $\{\sigma, \varepsilon_a - \varepsilon_m, \varepsilon_s - \varepsilon_m\}$



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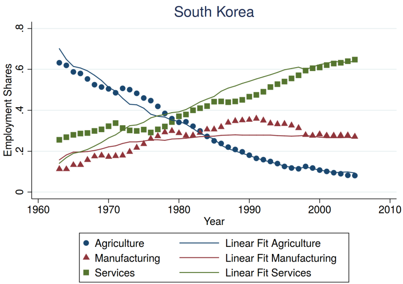

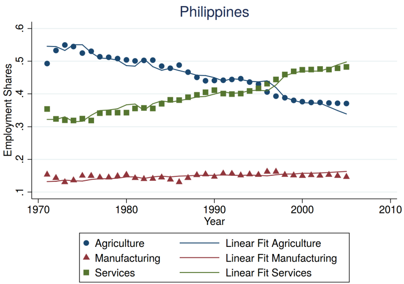

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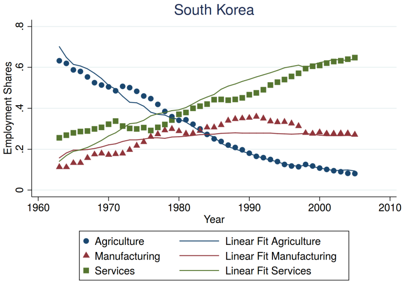
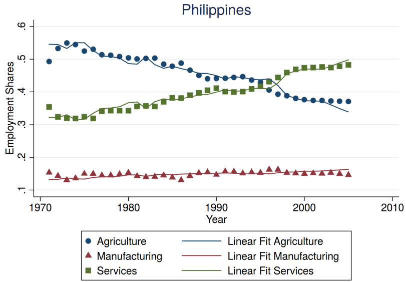
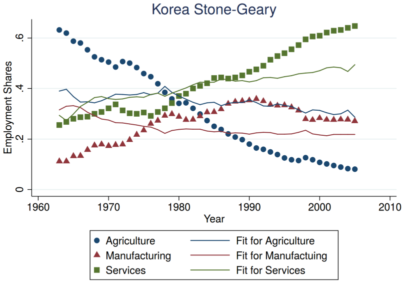
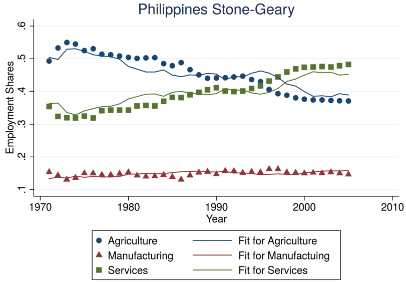


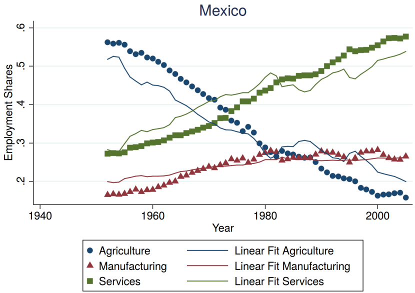


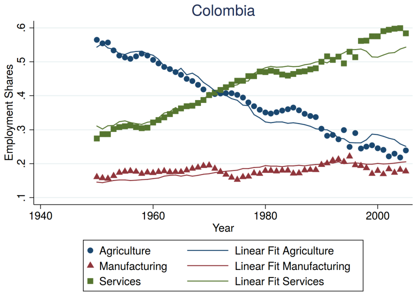




Latin America

Uses World Estimates for All Elasticities, $\{\sigma, \varepsilon_a - \varepsilon_m, \varepsilon_s - \varepsilon_m\}$





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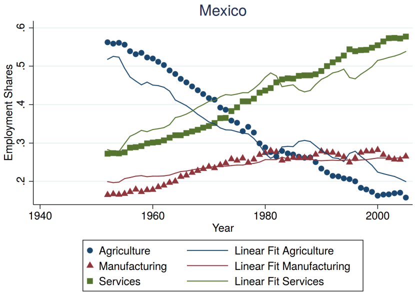
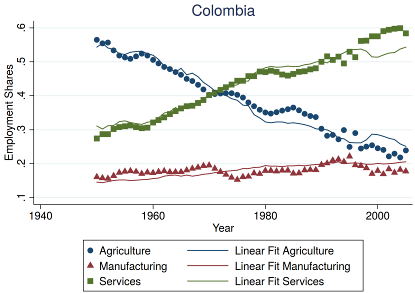
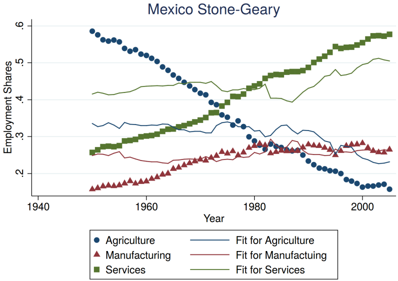
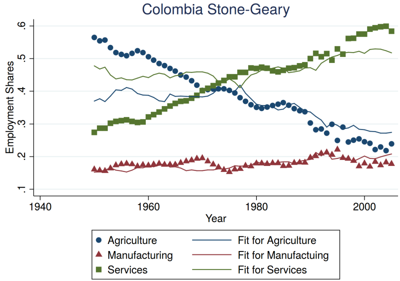


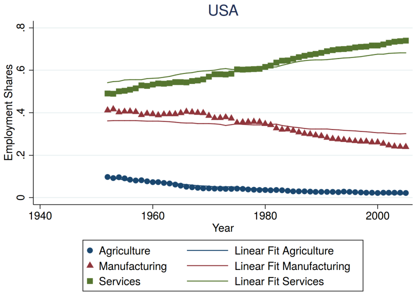


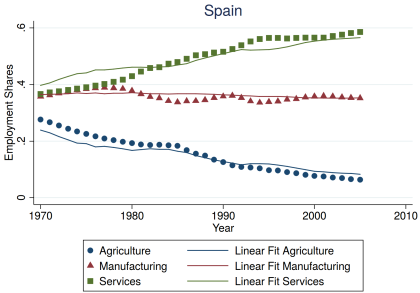




OECD*

Uses World Estimates for All Elasticities, $\{\sigma, \varepsilon_a - \varepsilon_m, \varepsilon_s - \varepsilon_m\}$





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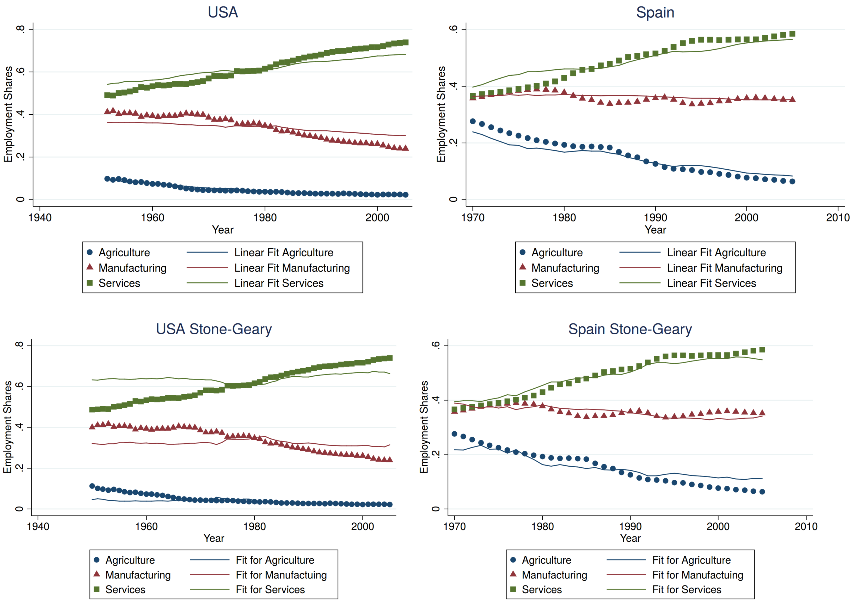

Contribution of Income and Prices

- % Variation Accounted by Income Effects in median year
  - 86% for Agriculture,
  - 57% for Manufacturing,
  - 82% for Services.

Model Specification Tests ▸ Partial Correlations

Specification	Log-Likelihood	LR Test χ^2	LR Test p-value	AIC	BIC
FE Only	-567.53	–	–	1235.05	1480.74
FE + Prices	-531.59	71.87	0.00	1165.18	1415.78
FE + Cons.	334.24	1803.53	0.00	-564.48	-308.97
Full Model	379.60	1894.25 90.72	0.00 0.00	-653.20	-392.77

Correlation Real and Nominal VA*

- Homothetic Pref. $+ \; 0 < \sigma < 1 \Rightarrow$ negative corr. real nominal,

$$\frac{p_{at} c_{at}}{p_{mt} c_{mt}} = \left(\frac{p_{at}}{p_{mt}}\right)^{(1-\sigma)} \quad \text{vs.} \quad \frac{c_{at}}{c_{mt}} = \left(\frac{p_{at}}{p_{mt}}\right)^{-\sigma}.$$

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- Homothetic Pref. $+ 0 < \sigma < 1 \Rightarrow$ negative corr. real nominal,

- Our model generates co-movement of real and nominal.

$$\frac{p_{at} c_{at}}{p_{mt} c_{mt}} = C_t^{\epsilon_a - \epsilon_m} \left(\frac{p_{at}}{p_{mt}} \right)^{(1-\sigma)} \quad \text{vs.} \quad \frac{c_{at}}{c_{mt}} = C_t^{\epsilon_a - \epsilon_m} \left(\frac{p_{at}}{p_{mt}} \right)^{-\sigma}.$$

- Positive correlation driven by income effects.
- Not targeted in the estimation.

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- Positive correlation driven by income effects.
- Not targeted in the estimation.

	Correlation	
	Data	Model
Agriculture/Manufacturing	0.95	0.93
Services/Manufacturing	0.80	0.71

Note: Results generated using World Estimates for all elasticities $\{\sigma, \varepsilon_a - \varepsilon_m, \varepsilon_s - \varepsilon_m\}$

Useful Properties NhCES not used in CLM

- Closed form representation Expenditure
- Aggregation across heterogeneous agents
- Logit micro-foundation
- Define the Cobb-Douglas Limit: extremely tractable and useful conceptually.
  - Corresponds to limit $\sigma \to 1$ in CLM.
  - We will see it in next class.

Closed Form

- Look at a rendition with a continuum of goods
- In this case we have that

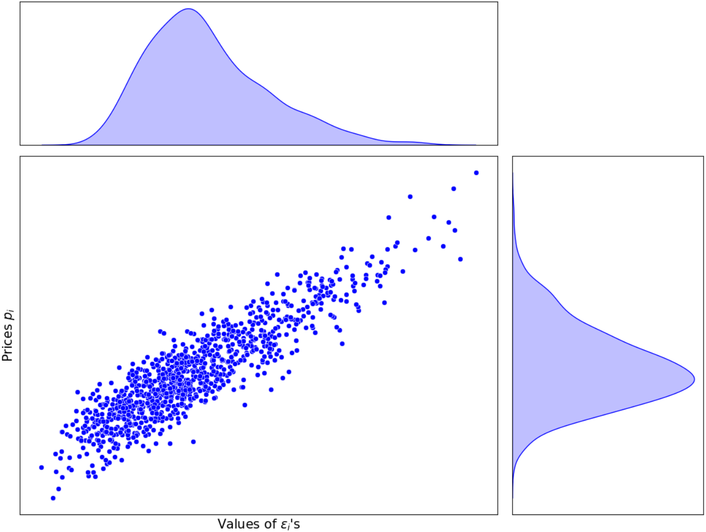
$$1 = \int_0^1 \left(\Omega_i U^{-\varepsilon_i} C_i\right)^{\frac{\rho-1}{\rho}} di, \tag{23}$$

$$C_i = \left(\frac{p_i}{E}\right)^{-\rho} \left[\Omega_i U^{-\varepsilon_i}\right]^{\rho-1}, \tag{24}$$

$$E = \left[\int_0^1 \left(\frac{p_i}{\Omega_i} U^{\varepsilon_i}\right)^{1-\rho} di\right]^{\frac{1}{1-\rho}}. \tag{25}$$

- Suppose $\ln p_i = \xi_p \varepsilon_i + \nu_p$, and $\ln \Omega_i = \xi_\Omega \varepsilon_i + \nu_\Omega$, where ν_p and ν_Ω can follow any distribution as long as the expectation $\mathbb{E}\left[\left(\frac{e^{\nu_p}}{e^{\nu_\Omega}}\right)^{1-\rho}\right]$ exists.

Example of case for closed form representation



Closed form ct'd

- Suppose that $\{\varepsilon_i\}_{i\in[0,1]}$ are distributed following a gamma distribution,

$$\varepsilon_i \sim \text{Gamma}(\alpha, \beta), \tag{26}$$

  where $\alpha > 0$ and $\beta > 0$ are the shape and scale parameters of the gamma distribution.

- Then we following closed form representation holds:

$$\ln U = \frac{\Upsilon}{1-\rho} - \frac{\Psi}{1-\rho} E^{-\frac{1-\rho}{\alpha}} \tag{27}$$

  where, $\Upsilon = \frac{1}{\beta} - (1-\rho)\left(\xi_p - \xi_\Omega\right) \in \mathbb{R}$
  $\Psi = \frac{\mathcal{M}^{\frac{1}{\alpha}}}{\beta} \in \mathbb{R}_+ \mathcal{M} \equiv \mathbb{E}\left[\left(\frac{e^{\nu_p}}{e^{\nu_\Omega}}\right)^{1-\rho}\right].$

- Consumption choices C_i in equation (24) are invariant to the scaling of β. Thus, we can normalize $\beta = 1$ wlog.

Aggregation across heterogeneous HH

- Consider household h with total expenditure E_h given by closed form Equation (27).
- The expenditure share in good i is

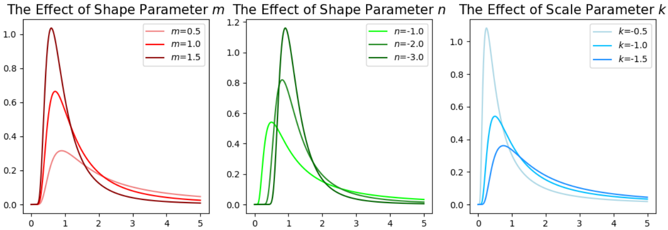
$$s_{ih} = \exp(\varepsilon_i \Upsilon) \left(\frac{\Omega_i}{p_i} \right)^{\rho-1} E_h^{\rho-1} \exp\left(-\varepsilon_i \Psi E_h^{\frac{\rho-1}{\alpha}} \right). \tag{28}$$

- Assume the expenditure distribution follows an Amoroso distribution, whose probability density function is given by,

$$f_{E_h}(x \mid l, k, m, n) = \frac{1}{\Gamma(m)} \left| \frac{n}{k} \right| \left(\frac{x-l}{k} \right)^{mn-1} \exp\left\{ - \left(\frac{x-l}{k} \right)^n \right\}. \tag{29}$$

  with $m > 0$ and $k, l, n \in \mathbb{R}$.
- Focus on the economically relevant case $k > 0$. In this case, the support of the distribution is $x \geq l$.
- Suppose the second shape parameter n of the Amoroso distribution (29) satisfies $n = \frac{\rho-1}{\alpha}$. The location parameter can be set to zero, $l = 0$ without loss of generality.

Amoroso distribution Examples



Aggregation ct'd

- Consider an economy populated by a continuum of agents indexed by h, whose total expenditure, E_h, is distributed Amoroso (Equation 29). Total expenditure shares of good i, $s_i = \int s_{ih} f_{E_h}(E_h) dE_h$ is

$$s_i = \exp(\varepsilon_i \Upsilon) \left(\frac{\Omega_i}{p_i}\right)^{\rho-1} \frac{\Gamma(m+\alpha)}{\Gamma(m)} \frac{k^{\rho-1}}{\left[1+\varepsilon_i \Psi k^{\frac{\rho-1}{\alpha}}\right]^{m+\alpha}}. \quad (30)$$

- The expenditure share in terms of the average household expenditure, $\bar{E}_h = k \frac{\Gamma\left(m+\frac{\alpha}{\rho-1}\right)}{\Gamma(m)}$, is $s_i = \int s_{ih} f_{E_h}(E_h) dE_h$

$$s_i = \exp(\varepsilon_i \Upsilon) \left(\frac{\Omega_i}{p_i}\right)^{\rho-1} \frac{\Gamma(m+\alpha)}{\Gamma(m+\frac{\alpha}{\rho-1})} \frac{k^{\rho-2}}{\left[1+\varepsilon_i \Psi k^{\frac{\rho-1}{\alpha}}\right]^{m+\alpha}} \bar{E}_h \quad (31)$$

Logit Microfoundation

- See note by Bohr Mestieri and Yavuz.

Application of NhCES to EK and AA

- Trade: Seamless integration with EK
  - Isoelasticity of prices allows for same derivations as in EK
- Spatial: Can trivially extend AA to allow for NhCES
  - AA strategy to proof uniqueness does not go through
  - Special Case of Interest: Heterothetic Cobb Douglas (Bohr, Mestieri and Robert-Nicoud)

Beyond NhCES: CREIS Preferences (Hanoch, 1975)

- Set Ω of goods.
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or, multilplying by C,

$$C = \int_{\omega \in \Omega} \underbrace{\zeta(\omega)^{\frac{1}{\sigma(\omega)}} C^{1-\epsilon(\omega)\frac{\sigma(\omega)-1}{\sigma(\omega)}}}_{\text{Weight}(C,\epsilon(\omega),\sigma(\omega),\zeta(\omega))} c(\omega)^{\frac{\sigma(\omega)-1}{\sigma(\omega)}} d\omega,$$

with $\sigma(\omega) > 1$ or $0 < \sigma(\omega) < 1$, $\zeta(\omega), \epsilon(\omega) > 0$, $\forall \omega \in \Omega$.

Beyond NhCES: CREIS Preferences (Hanoch, 1975)

- Set Ω of goods.
- Consumption of each good $c(\omega)$ for $\omega \in \Omega$.
- Consumption aggregator C implicitly defined as

$$1 = \int_{\omega \in \Omega} \zeta(\omega)^{\frac{1}{\sigma(\omega)}} \left(\frac{c(\omega)}{C^{\epsilon(\omega)}} \right)^{\frac{\sigma(\omega)-1}{\sigma(\omega)}} d\omega, \tag{32}$$

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with $\sigma(\omega) > 1$ or $0 < \sigma(\omega) < 1$, $\zeta(\omega), \epsilon(\omega) > 0$, $\forall \omega \in \Omega$.

- $c(\omega) \propto p(\omega)^{-\sigma(\omega)} C^{\varepsilon(\omega)} \to$ Flexible $\{\sigma(\omega), \epsilon(\omega)\}_{\omega \in \Omega}$.

Particular cases: Homothetic Demand

- Homothetic CES, $\epsilon(\omega) = 1$, $\sigma(\omega) = \sigma$

$$\int_{\omega \in \Omega} \zeta(\omega)^{\frac{1}{\sigma}} \left(\frac{c(\omega)}{C} \right)^{\frac{\sigma - 1}{\sigma}} d\omega = 1.$$

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  - CRS $\Rightarrow$ Suitable for modeling also production (zero profits).
  - Relative to nested CES, ES different across all inputs.
  - Example: Production Y with three skill levels $\{L, M, H\}$:

$$\left(\frac{L}{Y} \right)^{\frac{\sigma_L - 1}{\sigma_L}} + \left(\frac{M}{Y} \right)^{\frac{\sigma_M - 1}{\sigma_M}} + \left(\frac{H}{Y} \right)^{\frac{\sigma_H - 1}{\sigma_H}} = 1. \tag{34}$$

Particular cases: Non-Homothetic Demand

- Non-homothetic, Common Price Elasticities, $\sigma =$ constant

$$\int_{\omega\in\Omega} \zeta(\omega)^{\frac{1}{\sigma}} \left(\frac{c(\omega)}{C^{\epsilon(\omega)}} \right)^{\frac{\sigma-1}{\sigma}} d\omega = 1. \qquad (35)$$

- Constant ES, heterogeneous $\epsilon(\omega)$.
- Use this specification in application to Structural Change.

Particular Cases: Non-homothetic Demand, CRIE

- Start from

$$C = \int_{\omega \in \Omega} \zeta(\omega)^{\frac{1}{\sigma(\omega)}} C^{1-\epsilon(\omega)\frac{\sigma(\omega)-1}{\sigma(\omega)}} c\,(\omega)^{\frac{\sigma(\omega)-1}{\sigma(\omega)}}\, d\omega,$$

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- CRIE used in Fieler (11) and Caron et al. (14).
- $\sigma(\omega)$: price & income elasticity.
- Lashkari and Mestieri (2016) develop this further (hopefully a paper one day)