Market Size in General Equilibrium: Directed Technical Change and Home Market Effects

Martí Mestieri

Motivation

- Goal: Present theory for direction of innovation and the pattern of specialization.
- Same economic force: **Market Size**.
- Intuition: profits are larger the larger the market$\rightarrow$more firm entry.
- Will present a idiosyncratic view since this is last lecture :)
- Start discussing direction of innovation in long-run, facts and model. Then home market effect.

Two Views on Structural Change

1. Nonohomtheticity in preferences.
2. Differential technological progress across sectors.

- **Goal**: Combine two views endogenizing innovation process.
  - Endogenous direction of innovation across sectors/directed technical change.
  - Use non-homothetic demand system consistent with Engel curves not asymptoting to 1 (homotheticity).

Core Mechanism

- Relative demand:

$$\frac{Y_s}{Y_m} = \widetilde{\mathfrak{D}}\left(\frac{P_s}{P_m};\ C_{tot}\right)$$

- Relative prices/technology:

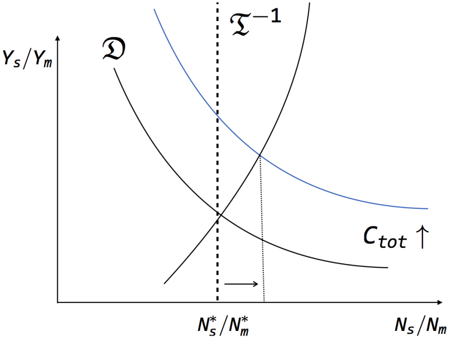
$$\frac{P_s}{P_m} = \mathfrak{P}\left(\frac{N_s}{N_m}\right)$$

$$\Rightarrow \frac{Y_s}{Y_m} = \mathfrak{D}\left(\frac{N_s}{N_m};\ C_{tot}\right)$$

- Technology supply:

$$\frac{N_s}{N_m} = \mathfrak{T}\left(\frac{Y_s}{Y_m}\right)$$

- Sector s more income elastic



Key Elements of First part of Lecture

- Document structural transformation in innovation
  - Use long-run evidence from patents
- Document income elasticities of US industry outputs correlated with
  - Rates of growth of patenting
  - Rates of growth of R&D expenditure
- Construct multisector growth model with
  1. Nonhomothetic CES demand
  2. Intersectoral knowledge spillovers
  3. Endogenous sectoral productivity growth
- Show the equilibria asymptotically predict correlation between income elasticity and innovation growth

Another Prominent Application: Biased Technical Change

- Modern literature on biased technical change:

  Autor et al. (1998); Acemoglu (1998, 2002, 2007)

  - Assume aggregate production function in aggregate factor inputs
  - Study response of factor-augmenting technology to shock in relative factor inputs?

    However, in LR, factor supply endogenous?

Outline

1. Reduced form Evidence on Income Elasticities.
   - Universe of U.S. patents.
     - Berkes (2017): Universe US patents.
     - USPTO: 1976 - onwards
   - U.S. Census of manufacturers.
2. Model
3. Home Market Effects

Reduced form Evidence on Role of Income Elasticity

- Do more income elastic sectors have more innovation?
- Historical Data: Universe US Patents Berkes 2017
- Last three decades: Two proxies for innovation
  - U.S. Patents $\longrightarrow$ Universe 1978 - 2014.
  - R&D expenditure $\longrightarrow$ U.S. Census Data + Compustat.
- For Income Elasticity
  - Structural Estimates using Nonhomothetic-CES.
  - Robust to Aguiar and Bils elasticities estimates from CEX.

Historical Evidence – Berkes (2017)

- Digitize *all* US patents, 1830 to 2016, from three sources.
- Use algorithm to identify citations pre-1940 citations (as they are in text).
    - We also have each patent geo-localized (not today).
- We identify the leading technological classes in each year of the sample as the most represented class in the top 10% patents in terms of forward citations in that year.

Data Collection

- Digitalized and OCR'ed all patents issued by USPTO into text.
- Used redundant external sources as checks.
  - USPTO digitalized patents.
  - Google Patents (and Maps).
  - Local repositories (e.g. Wyoming Inventor's Database)
  - HistPat.

Patent Examples

UNITED STATES PATENT OFFICE.

WM. F. GOODWIN, OF NEW YORK, N. Y.

IMPROVEMENT IN MOUNTING HAND-MORTARS.

Specification forming part of Letters Patent No. 46,101, dated January 31, 1865.

To all whom it may concern:

Be it known that I, WILLIAM F. GOODWIN, of the city, county, and State of New York, have invented a new and useful Improvement in Mortars; and I do hereby declare that the following is a full, clear, and exact description thereof, which will enable others skilled in the art to make and use the same, reference being had to the accompanying drawings, forming part of this specification, in which—

Figure 1 is a longitudinal axial section of a mortar constructed according to my invention. Fig. 2 is a like section on a larger scale, a part of the stake upon which the mortar is mounted being broken away. Fig. 3 is a side view from the opposite side to that presented in the other figures.

Similar letters of reference indicate like parts.

This invention consists, among other things, in mounting a mortar upon one end of a stake of wood or other suitable material, the other end of which is made pointed to enable one to insert it in the ground.

A is the mortar, formed with a powder-chamber, K, in its bottom. A cone, *d*, rising from the bottom of the powder-chamber, communicates through a channel, *e*, with the nipple.

B is a stout sleeve extending from the base of the mortar, to enable it to be attached to a stake, F, as shown in the drawings.

C is an elastic cushion, formed of rubber or equivalent material, placed at the bottom of the sleeve, so as to bear directly against the end of the stake. A slot, E, is made through the stake in that part covered by the sleeve, and it receives a pin, D, which passes through and is secured in the sides of the sleeve, so that when the mortar is fired it may slide longitudinally upon the stake, and yet be prevented from becoming displaced or being torn from the stake. The opposite end of the stake is shod with a pointed metallic ferrule or shoe, G, to enable it to be placed in the ground with facility.

H represents a lock, whose hammer-piece *f* is in outline the arc of a circle, and has a groove formed on its front side, which affords a path or channel for the passage upward of the gases which arise from the explosion of the cap, so that the lock shall not become injured thereby. The trigger *b* and sere *a* are operated by means of a cord or chain, (shown in red,) and the hammer is thrown down by the usual spring, J, inclosed in the lock-frame I, fast on one side of the sleeve B. A small hollow cylinder, O, closed at its rear end, is fastened on the right-hand side of the lock-frame, as seen in Fig. 3. A spiral spring in its bottom throws a small bolt, *c*, outward against the end of the trigger *b*, so as to restore the sere *a* to its place against the tumbler after every pull of the chain on the trigger, the latter resting against the end of the bolt when the hammer is cocked, as shown in that figure. The bolt will be retained in the cylinder between the spring and trigger by their mutual pressure against it.

The mortar may be made of bronze or any other suitable material. The stake or other support upon which it is mounted is to be portable, so that the mortar can be easily transported and fixed in the ground, or otherwise temporarily but firmly secured in a suitable position and inclination for the proper and efficient use of the weapon after the usual manner of using mortars. The axis of the stake upon which the mortar is mounted is coincident with that of the mortar; or, if not made coincident, they are always to be in parallel planes.

I am aware of the Letters Patent No. 43,881, granted August 16, 1864, to Ralph Graham, of Brooklyn, Kings county, New York, for a hand fire-arm adapted to projecting grenades or small bombs, and I do not claim the invention therein shown; but

What I do claim as new and of my invention, and for which I desire Letters Patent, is—

1. Constructing a mortar with a hollow sleeve projecting from its base, instead of trunnions or cheeks, substantially as above described, for the purpose of receiving the elastic cushion, or any equivalent spring, and the end of a stake, as above set forth.

2. The combination of the slot E and pin D with the aforesaid mortar A, sleeve B, and spring C, as and for the purposes specified.

WM. F. GOODWIN.

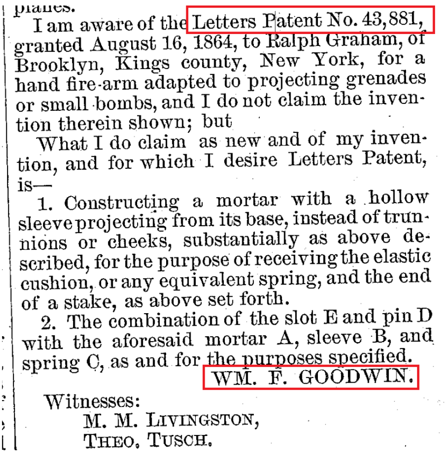
Witnesses:

M. M. LIVINGSTON,

THEO. TUSCH.

Patent Examples

Citations before 1947 and name.



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Patent Examples

Names, location, dates.

UNITED STATES PATENT OFFICE.

SPENCER LEE FRASER AND WILLIAM A. BRIGHAM, OF TOLEDO, OHIO.

OYSTER-REFRIGERATOR.

SPECIFICATION forming part of Letters Patent No. 300,061, dated June 10, 1884.

Application filed October 12, 1883. (No model.)

To all whom it may concern:

Be it known that we, SPENCER LEE FRASER and WILLIAM A. BRIGHAM, of Toledo, in the county of Lucas and State of Ohio, have invented certain new and useful Improvements

for the receptacle B. When access is desired to the receptacle for the removal of its contents, it is only necessary to remove the cover G, the ice in the box *a* being thus at all times covered and not exposed to the air at any

Patent Examples

Reference list after 1947.

material in the openings in said second block is exposed.

LAYTON R. FETTEROLF.

REFERENCES CITED

The following references are of record in the file of this patent:

UNITED STATES PATENTS

Number	Name	Date
723,258	Felton	Mar. 24, 1903
819,900	Martin	May 8, 1906
1,088,571	Heferman	Feb. 24, 1914
1,154,490	Davis	Sept. 21, 1915
1,504,326	Cullinan	Aug. 12, 1924
1,664,257	McCullough	Mar. 27, 1928
1,943,399	Smith	Jan. 16, 1934
1,968,626	Young	July 31, 1934
1,982,526	Lussky	Nov. 27, 1934
2,046,164	Herkner	June 30, 1936

FOREIGN PATENTS

Number	Country	Date
18,134	Great Britain	1902
431,884	Great Britain	July 17, 1935

Leading Technology Classes

- **Period 1830-1876**:
  1. Agriculture; Forestry; Animal Husbandry; Hunting; Trapping; Fishing
  2. Heating; Ranges; Ventilating
- **Period 1877-1958**:
  1. Engineering Elements or Units; General Measures for Producing and Maintaining Effective Functioning of Machines or Installations; Thermal Insulation in General
- **Period 1959-1969**:
  1. Conveying; Packing; Storing Handling Thin or Filamentary Material
  2. Organic Chemistry

Leading Technology Classes

- **Period 1976-1983**:
  1. Measuring; Testing
- **Period 1984-1995**:
  1. Medical or Veterinary Science; Hygiene
- **Period 1996-Present**:
  1. Computing; Calculating; Counting

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  1. Computing; Calculating; Counting

- Consistent with a picture where demand for different items was very different in 1830 than now.
  - Market size of agricultural products has declined, while it has increased for computing...

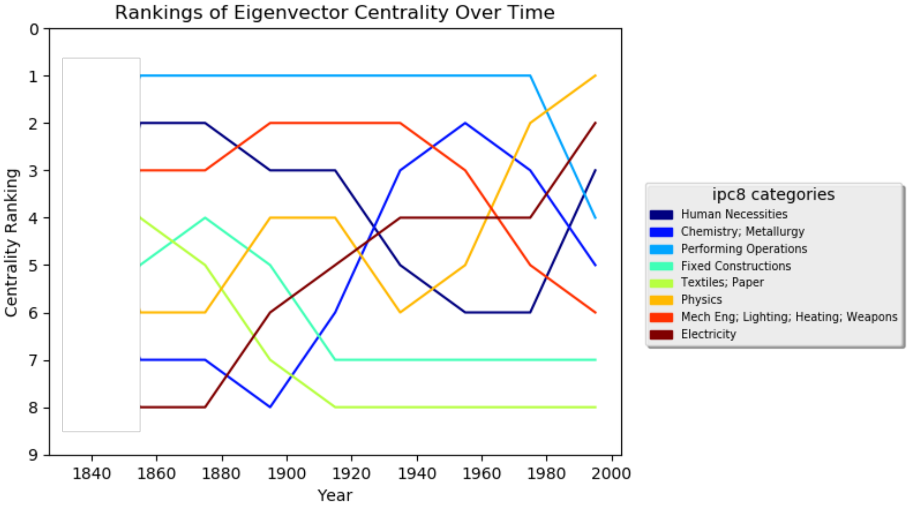
Leading Patent Categories

Most cited Category in Top 10% Patents

Years	IPC Class	Description
1836–1871	F24	Heating; Ranges; Ventilating
	A01	Agriculture; Forestry; Animal Husbandry; Hunting; Trapping; Fishing
1872-75	F16	Engineering Elements or Units; General Measures for Producing and Maintaining Effective Functioning of Machines or Installations; Thermal Insulation in General
1876	A01	Agriculture; Forestry; Animal Husbandry; Hunting; Trapping; Fishing
1877–1958	F16	Engineering Elements or Units; General Measures for Producing and Maintaining Effective Functioning of Machines or Installations; Thermal Insulation in General
1959–65	B65	Conveying; Packing; Storing Handling Thin or Filamentary Material
1966–67	C07	Organic Chemistry
1968–69	B65	Conveying; Packing; Storing Handling Thin or Filamentary Material
1970–75	C07	Organic Chemistry
1976–83	G01	Measuring; Testing
1984–95	A61	Medical or Veterinary Science; Hygiene
1996–present	G06	Computing; Calculating; Counting

Eigenvector Centrality

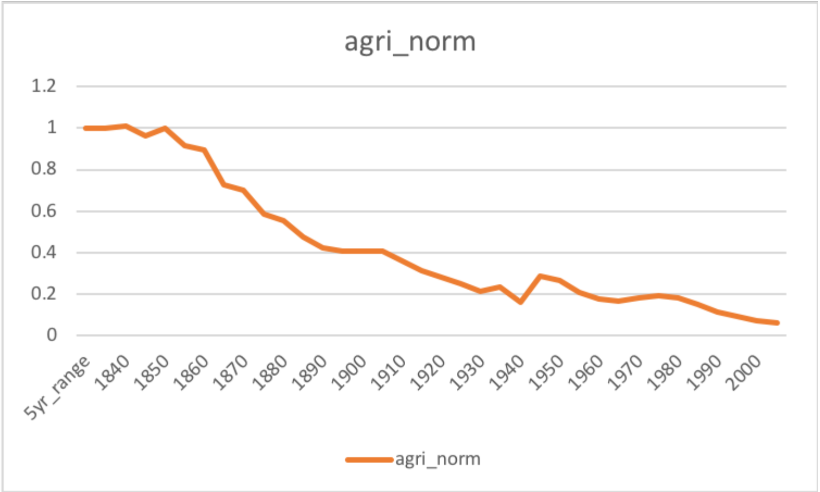
- Use citations across different patent classes.



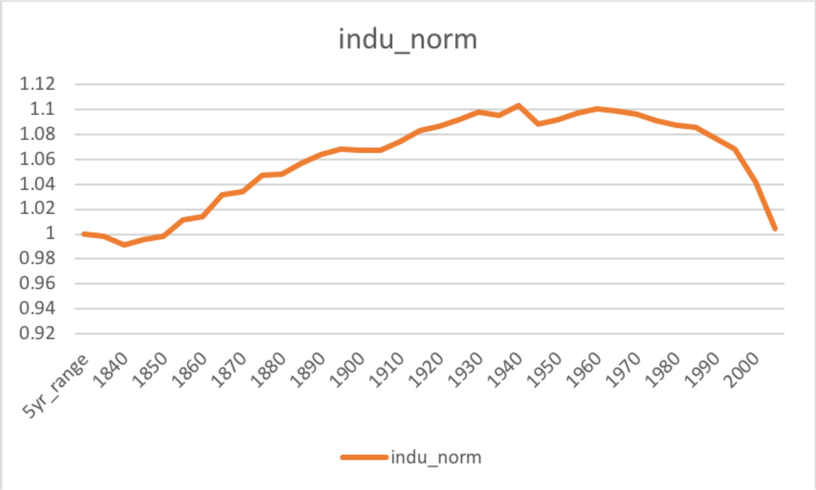
Evolution by Three Broad Sectors (CLM2)

- Assign each patent category to
  1. Agriculture
  2. Manufacturing
  3. Services
- Plot evolution normalizing initial level to 1.

Evolution by Three Broad Sectors (CLM2)



Evolution by Three Broad Sectors (CLM2)



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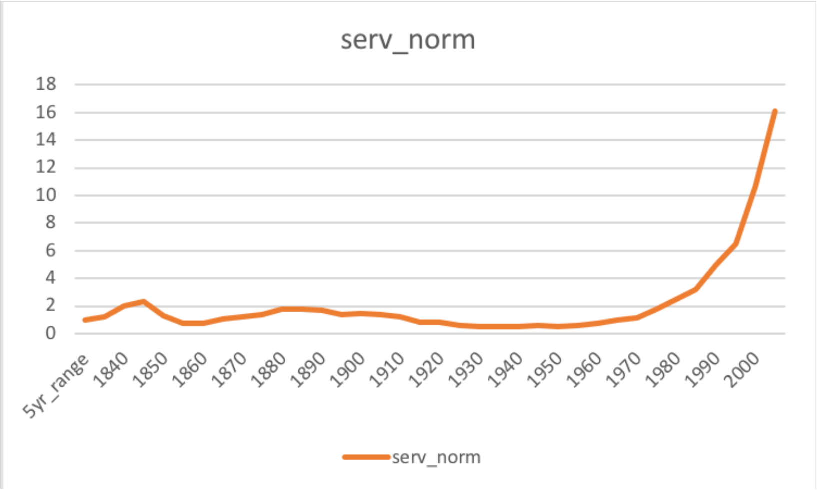
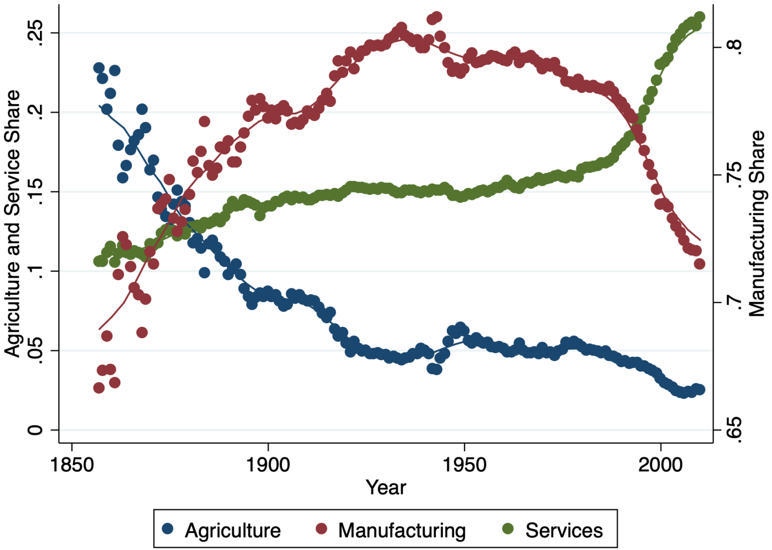


Evolution by Three Broad Sectors (CLM2)

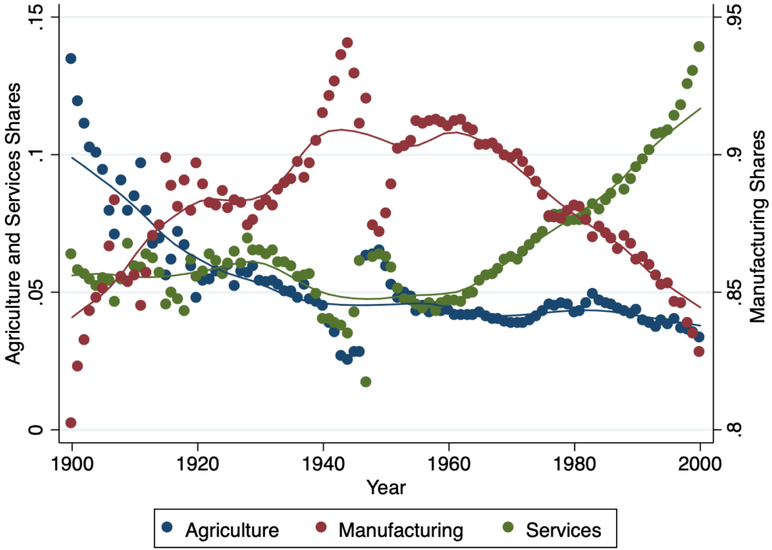
Sectoral Share of Innovation (over the 5-years period)

	Min (Year)	Average	Max (Year)
Agriculture	0.7% (2015)	4.5%	11% (1835)
Industry	89% (1840, 2015)	94.1%	98% (1945)
Services	<0.1% (<1950)	1.4%	10% (2015)

Three Sectors Together - US

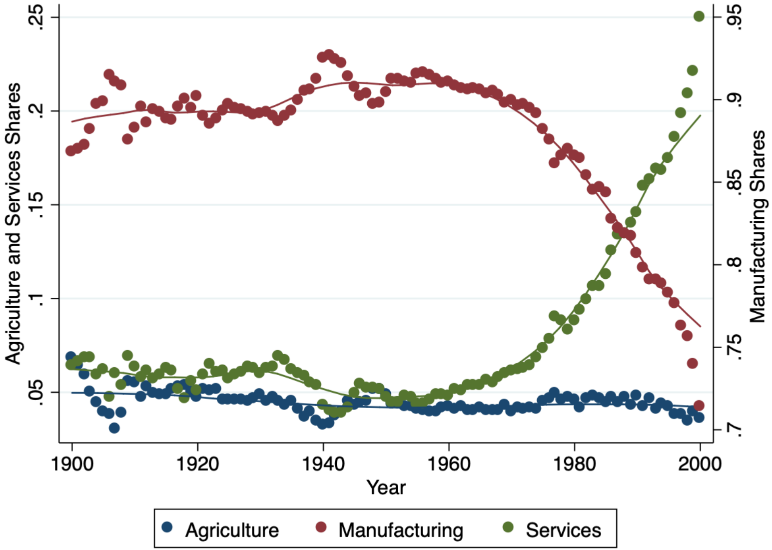


Germany

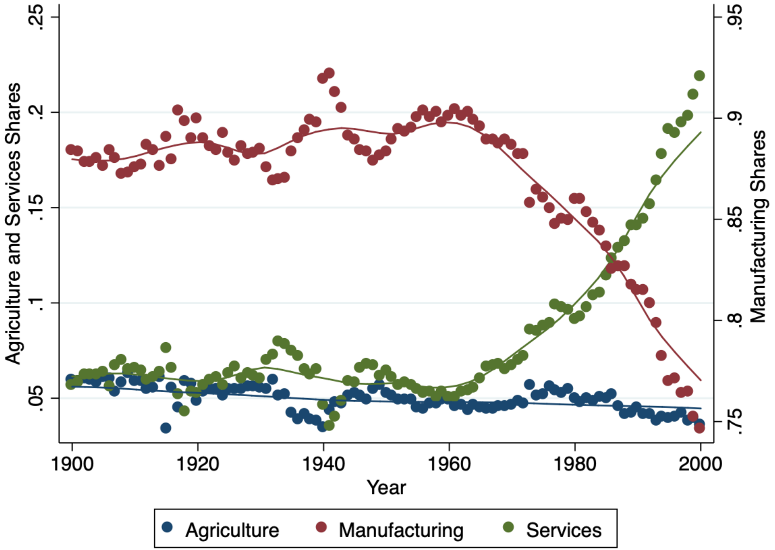


Agriculture and Services Shares
0
.05
.1
.15
Manufacturing Shares
.8
.85
.9
.95
1900
1920
1940
1960
1980
2000
Year
Agriculture
Manufacturing
Services

Great Britain

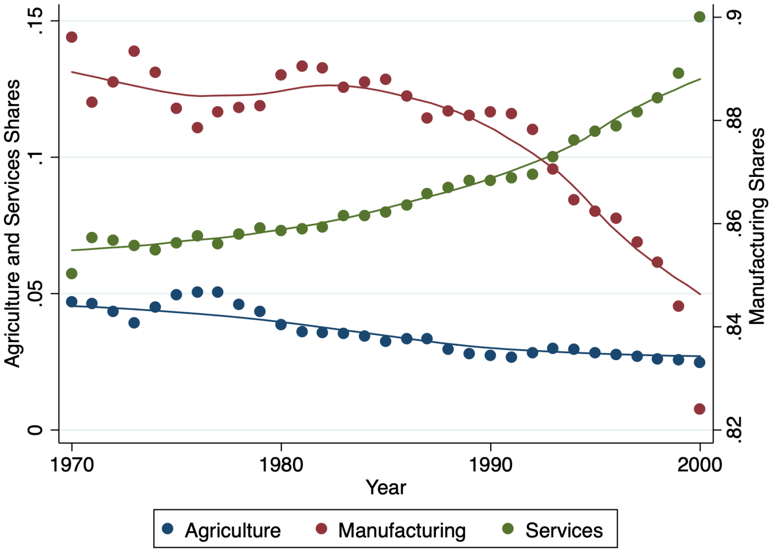


France



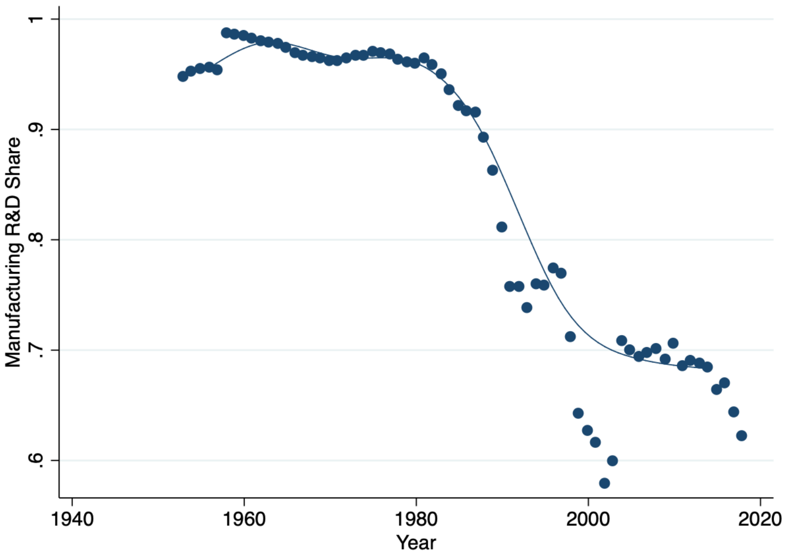
Agriculture and Services Shares
.05
.1
.15
.2
.25
Manufacturing Shares
.75
.8
.85
.9
.95
1900
1920
1940
1960
1980
2000
Year
Agriculture
Manufacturing
Services

Japan

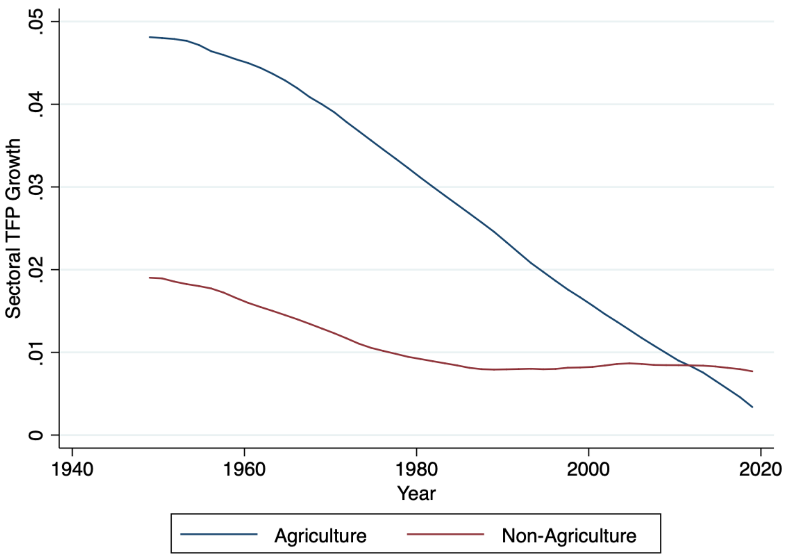


Agriculture and Services Shares
0
.05
.1
.15
Manufacturing Shares
.82
.84
.86
.88
.9
1970
1980
1990
2000
Year
Agriculture
Manufacturing
Services

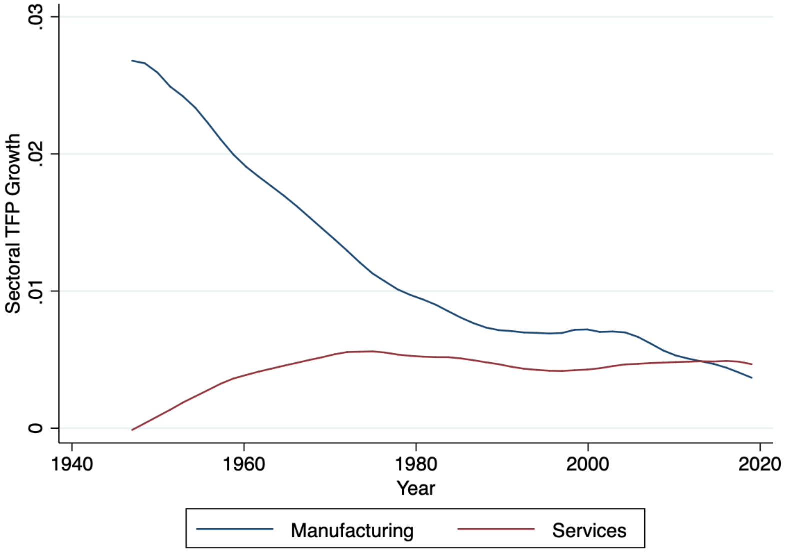
R& D US: Share in Manufacturing



TFP Growth US: Farm vs Non Farm



TFP Growth US: Manufacturing vs Services



Reduced form Evidence on Role of Income Elasticity II

- Run the following type of regression

$$y_{it} = \alpha + \beta\varepsilon_i + \delta_t + \delta_I + \nu_{it},$$

where

  - y_{it} is growth in R&D and patents in sector i,
  - δ_t is time fixed effect,
  - δ_I broad sector FE (SIC 1 for R&D), (NAICS 1 patents).

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  where
  - y_{it} is growth in R&D and patents in sector i,
  - δ_t is time fixed effect,
  - δ_I broad sector FE (SIC 1 for R&D), (NAICS 1 patents).
- Median growth of patents in the sample: 0.024
- P90 growth 12%, P10 -13%
- Median income elasticity in the sample: 1.05
- P90 income elasticity 1.3, P10, 0.88

Results for Patents

$$\text{Yearly Patent Growth}_{it} = \alpha + \beta\varepsilon_i + \delta_t + \delta_{NAICS1} + \nu_{it},$$

	Raw Citations			Weighted Citations		
	(1)	(2)	(3)	(4)	(5)	(6)
Elasticity	0.024	.024***	.016***	.025	.025***	.016***
	(.020)	(.007)	(.007)	(.021)	(.006)	(.008)
Year FE	No	Yes	Yes	No	Yes	Yes
Ind. FE	No	No	Yes	No	No	Yes
R^2	.0004	.91	.91	.0005	.90	.90

Obs. 3002, s.e.: robust, clustered at year and NAICS 1, respectively.

Results for R&D Expenditure – Census of Manufacturers

$$\text{Yearly R\&D Growth}_{it} = \alpha + \beta\varepsilon_i + \delta_t + \delta_{SIC1} + \nu_{it},$$

	(1)	(2)	(3)
Elasticity	0.001	.136***	.496***
	(.069)	(.06)	(.127)
Year FE	No	Yes	Yes
Broad Ind. FE	No	No	Yes
R-squared	.004	.257	.349

Number obs. is 1120. Robust standard errors in parenthesis. Weighted regression by number of obs. by industry.

- Also holds for Compustat sample (actually looks better).

Theory: BGP and Endogenous Structural Change

Overview

Expanding Varieties Model of Endogenous Growth (Romer 1990)

- Households have preferences over products **within and across sectors**.
- Labor is used to produce existing goods or to innovate new varieties of goods.
- Firms are free to innovate **in any sector** and receive a perpetual patent on any new good they innovate.

Households Temporal Preferences

Mass L of identical households.

Households trade shares A_t in an aggregate portfolio of firms.

The household intertemporal utility and budget constraint:

$$\int_0^\infty \exp(-\delta t) \ln C_t dt$$

$$\dot{A}_t = r_t A_t + W_t - E_t,$$

- C_t - Instantaneous utility
- W_t - Wage rate
- E_t - Expenditures

Households Sectoral Preferences

Products across and within sectors are defined by the product space

$$(\varepsilon, i) \in [0, \infty) \times [0, N_{\varepsilon,t}]$$

- $N_{\varepsilon,t}$ - Number of distinct products in sector ε.

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Nonhomothetic CES over sectors and standard CES within sectors

$$1 = \int_0^\infty \left(C_t^{-\varepsilon} C_{\varepsilon,t}\right)^{\frac{\rho-1}{\rho}} d\varepsilon \qquad \text{and} \qquad C_{\varepsilon,t} = \left(\int_0^{N_{\varepsilon,t}} C_{\varepsilon i,t}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}$$

- $0 < \rho < 1 < \sigma$.
- C_t utility at time t, with $C_t \in (0, 1)$. More

Nonhomothetic CES separates Income and Price Effects

$$1 = \int_0^\infty \left(C_t^{-\varepsilon} C_{\varepsilon,t} \right)^{\frac{\rho-1}{\rho}} d\varepsilon$$

- $\rho \geq 0$ is the constant elasticity of substitution
- Price elasticity independent of expenditure elasticity (escape Pigou's law)

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- Price elasticity independent of expenditure elasticity (escape Pigou's law)
- $\varepsilon \geq 0$ parametrizes the strength of income effects (and indexes goods too)

$$C_t = \left(\int_0^\infty \left(C_t^{1-\varepsilon} C_{\varepsilon,t} \right)^{\frac{\rho-1}{\rho}} d\varepsilon \right)^{\frac{\rho}{\rho-1}}$$

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- All results extend to add constant taste parameter

$$C_t = \left(\int_0^\infty \left(\varepsilon^{-\beta} C_t^{1-\varepsilon} C_{\varepsilon,t} \right)^{\frac{\rho-1}{\rho}} d\varepsilon \right)^{\frac{\rho}{\rho-1}} \qquad \text{where} \qquad \beta \geq 0.$$

Nonhomothetic CES separates Income and Price Effects

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- $\varepsilon \geq 0$ parametrizes the strength of income effects (and indexes goods too)
- All results extend to add constant taste parameter
- Key: good is luxury or subsistence depending on level of expenditure

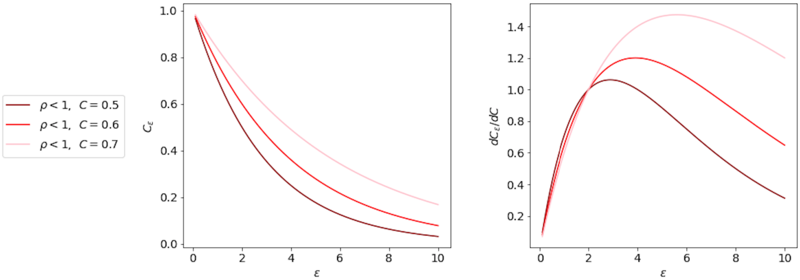
$$\eta_{\varepsilon,t} \equiv \frac{\partial \ln C_{\varepsilon,t}}{\partial \ln E_t} > 1 \quad \Longleftrightarrow \quad \varepsilon > \ \bar{\varepsilon}_t \equiv \int_0^\infty \frac{P_{\varepsilon,t} C_{\varepsilon_t}}{E_t} \varepsilon d\varepsilon$$

  - *Remark*: Rank order is constant, $\eta_{\varepsilon,t} > \eta_{\varepsilon',t} \Longleftrightarrow \varepsilon > \varepsilon'$.

Sectoral Relative Demand

$$C_\varepsilon = \left(\frac{P_\varepsilon}{E}\right)^{-\rho} C^{(1-\rho)\varepsilon}$$

$$\rho \in (0,1), \qquad C \in [0,1)$$



Household Sectoral Preferences Feature Two Nests

Products across and within sectors are defined by the product space

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Within sectors: CES preferences with $\sigma > 1$ (features love-for-variety effect)

$$C_{\varepsilon,t} = \left(\int_0^{N_{\varepsilon,t}} C_{\varepsilon i,t}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

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$$C_{\varepsilon,t} = \left(\int_0^{N_{\varepsilon,t}} C_{\varepsilon i,t}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

Note: Need to check monotonicity conditional on price-distribution and total expenditure (similar to other nonhomothetic demand systems like PIGL, Stone-Geary)

Heterothetic Cobb-Douglas: Isolates Income Effects

Across sectors we could have Heterothetic Cobb-Douglas with sectoral weights given by $\alpha_\varepsilon(C_t)$

$$\ln C_t = \int_0^\infty \alpha_\varepsilon(C_t) \ln\left(C_{\varepsilon,t}\right) d\varepsilon$$

Sectoral expenditure shares only depend on preference weights

$$\alpha_\varepsilon(C_t) = \frac{P_{\varepsilon,t} C_{\varepsilon,t}}{P_t C_t}$$

- ε ranks a sectors income elasticity with higher ε sectors being more income elastic.
- $\beta \geq 0$ controls the strength of the non-homotheticities.

Heterothetic Cobb-Douglas: Isolates Income Effects

Across sectors we could have Heterothetic Cobb-Douglas with sectoral weights given by $\alpha_\varepsilon(C_t)$

$$\ln C_t = \int_0^\infty \alpha_\varepsilon(C_t) \ln\left(C_{\varepsilon,t}\right) d\varepsilon$$

Sectoral expenditure shares only depend on preference weights

$$\alpha_\varepsilon(C_t) = \frac{P_{\varepsilon,t} C_{\varepsilon,t}}{P_t C_t} = C_t^{-\beta} \exp\left(-C_t^{-\beta} \varepsilon\right)$$

- ε ranks a sectors income elasticity with higher ε sectors being more income elastic.
- $\beta \geq 0$ controls the strength of the non-homotheticities.

Heterothetic Cobb-Douglas: Some Additional Properties

- Expenditure function:

$$\ln E = \ln C + \underbrace{\int_0^\infty \alpha_\varepsilon(C) \ln\left(\frac{P_\varepsilon}{\alpha_\varepsilon(C)}\right) d\varepsilon}_{\equiv \ln P}$$

- $\ln P$ coincides with homothetic Cobb-Douglas!
- Bohr, Mestieri and Robert-Nicoud 23 provide sufficient cond. for monotonicity.
  - Quasi-concavity for free.
- Also show how to use
  1. CD Price Index to recover money-metric cost of inflation with HH data.
  2. HCD to obtain Kongsamut et al. (2001) generically.
  3. HCD in spatial equilibrium: equalization of C implies same $\alpha_\epsilon(C)$ in space.

Production and Profits

Homogenous linear production technology in each sector:

$$Y_{\varepsilon i,t} = L_{\varepsilon i,t}.$$

Firms compete monopolistically and maximize profits:

$$P_{\varepsilon i,t} = \frac{\sigma}{\sigma - 1} W_t \qquad \text{and} \qquad \Pi_{\varepsilon i,t} = \frac{1}{\sigma - 1} W_t Y_{\varepsilon i,t}.$$

Value of Product:

$$V_{\varepsilon i,t} = \int_t^{\infty} \exp\left(- \int_t^s r(\tau) d\tau \right) \Pi_{\varepsilon i}(s) ds.$$

Innovation and Free Entry

Innovation technology in each sector:

$$\dot{N}_{\varepsilon,t} = \eta N_t^{\phi} L_{R\varepsilon,t}.$$

where $N_t = \int_0^{\infty} N_{\varepsilon,t} d\varepsilon$ is the total number of products and $\eta > 0$.

- Start with $\phi = 1$. Then show results extend to $\phi < 1$.
- If innovation was *not* directed but random across all ε, no relative price behavior.

Free and postive entry across all sectors:

$$\eta N_t V_{\varepsilon i,t} = W_t \quad \forall \varepsilon \in [0,\infty) \quad \Longrightarrow \quad V_{\varepsilon i,t} = V_t \quad \text{and} \quad \Pi_{\varepsilon i,t} = \Pi_t.$$

Definition of Equilibrium

1. Households Maximize Utility
2. Firms Maximize Profits
3. Free Entry Condition is Satisfied
4. Goods and labor markets clear

Equilibrium Product and Price Distribution

Profit and Free Entry Condition:

- Free entry$\rightarrow$ individual firms make same profit$\rightarrow$extensive margin only
- Total labor in production $L_{Y,t} = N_t L_{\varepsilon i,t}$
- Invert profit equation to obtain

$$N_{\varepsilon,t} = \left(\frac{LE_t}{\sigma \Pi_t} \left(\frac{L_{Y,t}}{L} \right)^{\rho-1} C_t^{\varepsilon(1-\rho)} \right)^{\frac{\sigma-1}{\sigma-\rho}}$$

Note: for HCD just set $\rho = 1$

Sectoral Price Index and Optimal Pricing:

- Combine CES price index $P_{\varepsilon,t}^{1-\sigma} = \int_0^{N_{\varepsilon,t}} P_{\varepsilon i,t}^{1-\sigma} di$ and symmetry of firm pricing:

$$P_{\varepsilon,t} = \frac{\sigma}{\sigma-1} W_t N_{\varepsilon,t}^{-\frac{1}{\sigma-1}}$$

We obtain a Closed-Form Utility to Expenditure Mapping

Expenditure Minimization of Nonhomotheic CES:

- Dual of $\max_{\{C_{\varepsilon,t}\}} C_t$ s.t. $\int_0^\infty P_{\varepsilon,t} C_{\varepsilon,t} \leq E_t$

$$E_t = \left(\int_0^\infty \left(C_t^\varepsilon P_{\varepsilon,t} \right)^{1-\rho} d\varepsilon \right)^{\frac{1}{1-\rho}}$$

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- Integrate using equilibrium product distribution $N_{\epsilon,t}$ in price index $P_{\varepsilon,t}$

$$\ln C_t = -\frac{\sigma-\rho}{(\sigma-1)(1-\rho)} \left(\frac{LE_t}{\sigma \Pi_t} \left(\frac{L_{Y,t}}{L} \right)^{\sigma-1} \right)^{-\frac{1-\rho}{\sigma-\rho}}$$

Equilibrium Product and Price Distribution Revisited

Product Distribution as a function of economic aggregates only:

$$N_{\varepsilon,t} = N_t \cdot \Psi_t \exp\left(-\Psi_t \varepsilon\right)$$

where $\Psi_t(N_t) = \left(\left(\frac{L_{Y,t}}{L}\right)^{\sigma-1} N_t\right)^{-\alpha}$, $N_t = \frac{LE_t}{\sigma \Pi_t}$, and $\alpha = \frac{1-\rho}{\sigma-\rho} \in (0,1)$

Equilibrium Product and Price Distribution Revisited

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- As N_t grows, slower decay in exponential distrib.$\rightarrow$more income elastic goods

Price Distribution mirrors product distribution:

$$P_{\varepsilon,t} = \underbrace{E_t \left(\frac{L_{Y,t}}{L}\right)^{-1} \left(N_t \Psi_t(N_t)\right)^{-\frac{1}{\sigma-1}}}_{\zeta_t} \exp\left(\underbrace{\frac{1}{\sigma-1}\Psi_t(N_t)}_{\chi_t} \cdot \varepsilon\right)$$

Household Euler Equation

Household Intertemporal Problem:

$$\max_{C_t, A_t} \quad \int_0^\infty e^{-\delta t} \ln C_t dt \qquad \text{s.t.} \qquad \dot{A}_t = r_t A_t + W_t - E_t$$

$$P_{\varepsilon,t} = \zeta_t \exp\left(\chi_t \cdot \varepsilon\right)$$

$$E_t = \left(\int_0^\infty \left(C_t^\varepsilon P_{\varepsilon,t}\right)^{1-\rho} d\varepsilon\right)^{\frac{1}{1-\rho}}$$

Solution given by:

$$\implies C_t = \exp\left(-\chi_t - \frac{\zeta_t^{1-\rho}}{1-\rho} E_t^{-(1-\rho)}\right)$$

$$\implies \frac{\dot{E}_t}{E_t} = \frac{1}{2-\rho}\left((r_t - \delta) + (1-\rho)\frac{\dot{\zeta}_t}{\zeta_t}\right)$$

Transversality Condition: $\lim_{t\to\infty} \exp\left(-\int_0^t r_s ds\right) N_t V_t = 0$ where $N_t V_t = A_t$

Balanced Growth Path: Preliminaries and Definition

1. **Define BGP by constant expenditure growth (or constant r_t)**
   - Convenient to express results in terms of $g_N \equiv \dot{N}/N$

2. **Ensure positive growth and satisfied transversality condition, assume:**

$$\frac{\eta}{\sigma - 1} L > \delta$$

3. **Normalize price-level** $\zeta_t = 1 \iff$ **Price of sector** $\varepsilon = 0$

$$P_{0,t} = 1$$

Aggregate Balanced Growth

∃! BGP with constant growth rates of wages, expenditures, and profits,

$$g_E = g_W = \frac{1}{\sigma - \rho} g_N, \qquad g_\Pi = g_V = \left(\frac{1}{\sigma - \rho} - 1 \right) g_N,$$

constant interest rate

$$r = \delta + \left(\frac{1}{\sigma - \rho} + \alpha \right) g_N,$$

constant labor in research and growth of products

$$g_N = \eta L_R = \frac{1}{1 + \alpha + \frac{1}{\sigma - 1}} \left(\frac{\eta L}{\sigma - 1} - \delta \right).$$

Economy features Unbalanced Sectoral Growth along the BGP

Sectoral dynamics:

$$N_{\varepsilon,t} = \left(\left(\frac{L_{Y,t}}{L}\right)^{\sigma-1} N_t\right)^{-\alpha} \exp\left(-\left(\left(\frac{L_{Y,t}}{L}\right)^{\sigma-1} N_t\right)^{-\alpha} \cdot \varepsilon\right) N_t,$$

$$\frac{\dot{N}_{\varepsilon,t}}{N_{\varepsilon,t}} = \left((1-\alpha) + \alpha\left(\left(\frac{L_{Y,t}}{L}\right)^{\sigma-1} N_t\right)^{-\alpha} \cdot \varepsilon\right) \frac{\dot{N}_t}{N_t}.$$

- **Along BGP**: $\frac{\dot{N}_t}{N_t} = g_N$ and $\frac{L_{Y,t}}{L} = \frac{L_Y}{L} \longrightarrow$ simple dynamics.
  - Sectoral dynamics can be described as a function of only N_t.
  - Always positive growth in all sectors, but at different rates.
- Heterogeneity in sectoral growth only through ε. Testable?

Connection with Empirical Motivation on Price and Patent Growth

- Prices inherit the "inverse" properties of $N_{\varepsilon,t}$ since $P_{\varepsilon,t} \propto N_{\varepsilon,t}^{-1/(\sigma-1)}$,

$$\frac{\dot{P}_{\varepsilon,t}}{P_{\varepsilon,t}} = -\frac{1}{\sigma-1}\frac{\dot{N}_{\varepsilon,t}}{N_{\varepsilon,t}} + \frac{\dot{W}_t}{W_t}$$

$\Longrightarrow$ Consistent with PCE price growth evidence from motivation.

- Proxy idea/product flow $\dot{N}_{\varepsilon,t}$ to patents and $W_t L_{R\varepsilon,t}$ to R&D expenditures
  - Along BGP sectoral dynamics imply:

$$\frac{\partial^2 \ln \dot{N}_{\varepsilon,t}}{\partial\varepsilon\partial t} > 0, \qquad \frac{\partial^2 \ln L_{R\varepsilon,t}}{\partial\varepsilon\partial t} > 0.$$

$\Longrightarrow$ Consistent with patent and R&D evidence from motivation.

Description of the Unbalanced Sectoral Growth along the BGP

- Along BGP, hump shaped pattern of $N_{\varepsilon,t}/N_t$.
- Sectoral output shares depend on relative product shares (times dampening)

$$\frac{P_{\varepsilon,t}C_{\varepsilon,t}}{P_{\varepsilon',t}C_{\varepsilon',t}} = \frac{N_{\varepsilon,t}}{N_{\varepsilon',t}} e^{-\kappa(\varepsilon-\varepsilon')N_t^{-\alpha}}$$

- Time of hump and (take off) is increasing in ε

$$t_\varepsilon^{hump} = \frac{\ln \varepsilon}{\alpha g_N} + \text{Constant.}$$

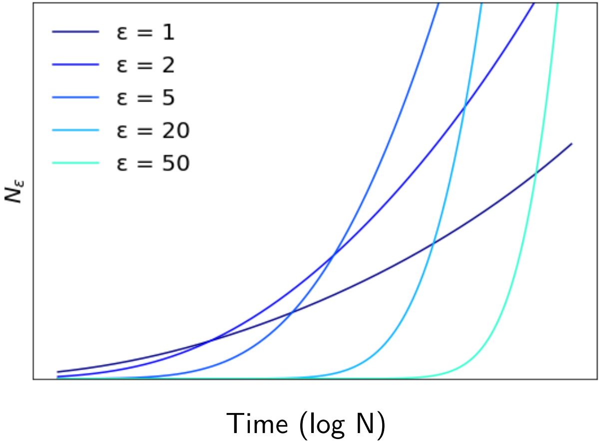
$\Longrightarrow$ Consistent with US sectoral peaks correlation with η_ε from motivation

$\Longrightarrow$ Consistent with cross-country correlation of sectoral peaks

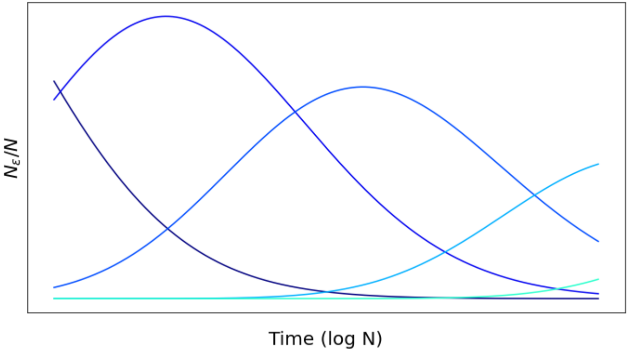
- Expenditure elasticity of good at peak declines in ε

$$\eta_\varepsilon^{hump} = \rho + (1-\rho)\left(\frac{\sigma-1}{\sigma}\right)^{2-\rho} \varepsilon^{-\frac{1}{1-\rho}}$$

Sectors Take off Sequentially following their Expenditure Elasticity



Sectoral Share is Hump-Shaped over Time



Perpetual Sectoral Turnover along the BGP

1. **A sector's expenditure elasticity declines as incomes grow:**

$$\eta_{\varepsilon}(C_t) = \frac{\partial \ln E_{\varepsilon,t}}{\partial \ln E_t} = (1-\beta) + \beta C_t^{-\beta} \epsilon$$

Perpetual Sectoral Turnover along the BGP

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$$\eta_\varepsilon(C_t) = \frac{\partial \ln E_{\varepsilon,t}}{\partial \ln E_t} = (1-\beta) + \beta C_t^{-\beta} \epsilon$$

2. Relative sectoral dynamics only depend on their expenditure elasticity:

$$\frac{\dot{N}_{\varepsilon,t}}{N_{\varepsilon,t}} = g_N \cdot \eta_\varepsilon(C_t)$$

- Always positive growth in all sectors + more expenditure elastic sectors grow faster.

Perpetual Sectoral Turnover along the BGP

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2. Relative sectoral dynamics only depend on their expenditure elasticity:

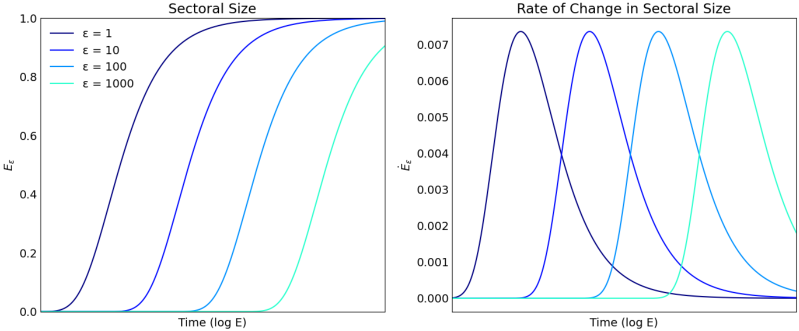
$$\frac{\dot{N}_{\varepsilon,t}}{N_{\varepsilon,t}} = g_N \cdot \eta_\varepsilon(C_t)$$

- Always positive growth in all sectors + more expenditure elastic sectors grow faster.

3. Sectors take off and peak in order of their expenditure elasticity rank ε:

- Sector peaks when its expenditure elasticity, $\eta_\varepsilon(C_t) = 1$ (which happens at $C_t = \varepsilon^{1/\beta}$).
- Income elastic sectors grow faster than aggregate; income inelastic sectors grow slower.

3. Sectors Take Off Sequentially following their Expenditure Elasticity



This behavior keeps going ad infinitum (aka "traveling wave")

▸ Sector shares

Social Planner Problem: Problem Set-up

$$\max_{\{L_{\varepsilon i,t}\},\{L_{R\varepsilon,t}\}} L\int_0^{\infty} \exp\left(-\delta t\right)\ln(C_t)dt$$

s.t.

$$1 = \left(\int_0^{\infty}\left(C_t^{-\varepsilon}C_{\varepsilon,t}\right)^{\frac{\rho-1}{\rho}}d\varepsilon\right)^{\frac{\rho}{\rho-1}}$$

$$C_{\varepsilon,t} = \left(\int_0^{N_{\varepsilon,t}} C_{\varepsilon i,t}^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}$$

$$Y_{\varepsilon i,t} = L_{\varepsilon i,t}$$

$$\dot{N}_{\varepsilon,t} = \eta N_t L_{R\varepsilon,t}$$

$$Y_{\varepsilon i,t} = LC_{\varepsilon i,t}$$

$$L_t = \int_0^{\infty}\int_0^{N_{\varepsilon,t}} L_{\varepsilon i,t}did\varepsilon + \int_0^{\infty} L_{R\varepsilon,t}d$$

▸ Social Planner Derivations ▸ Jump to Conclusion

Comparison to Decentralized Equilibrium BGP: Faster Growth in SP

Growth Rates of Social Planner and Decentralized Equilibrium

$$\text{SP}: \qquad g_N = \frac{1}{\alpha + \frac{1}{\sigma-1}} \left(\frac{\eta L}{\sigma - 1} - \delta \right)$$

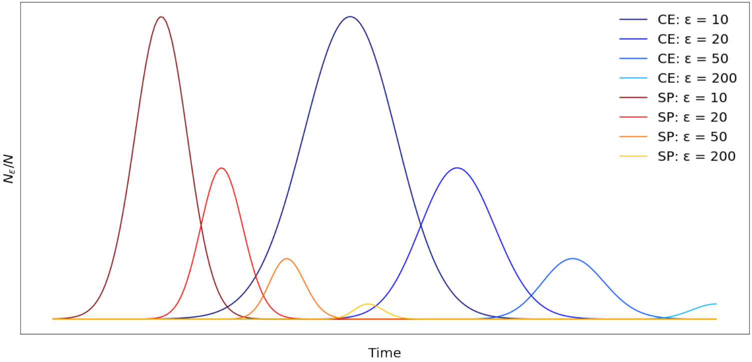
$$\text{DE}: \qquad g_N = \frac{1}{1 + \alpha + \frac{1}{\sigma-1}} \left(\frac{\eta L}{\sigma - 1} - \delta \right)$$

Externality: The same as with homothetic preferences. There is a difference in the level of R&D expenditures, but the relative allocations are "right"

Comparison of Sectoral Behaviour SP vs DE

- Take-off ordering and relative intensities are the same:
  - Markets get the right signals and allocate research correctly across sectors.
  - Reach same maximum in hump $N_{\varepsilon,t}/N_t$.
  - Timing of peak in DE is off, SP satisfies $\eta_{SP}^{hump} = 1$.
- Standard R&D subsidy implements SP allocation.
- Support for targeted industrial policy? Look elsewhere.
- Lesson for countries lagging behind: *not* do IP targeting leader tech. makeup.

Sectoral Behavior: SP vs DE



CEX - PCE Evidence on Price Growth: Data Construction

Compute expenditure elasticities for granular PCE categories

▸ Aguiar and Bils (2015)

- Use Consumer Expenditure Survey UCC categories (600+), 2000-2004
- Match with PCE disaggregated categories (150+)
  - BLS bridge + manual match

- TBD: structural Equation from HCD almost identical to Aguiar and Bils 15.

Compute average price category growth over 1959-2020 ▸ Examples

Regress average price growth against expenditure elasticities.

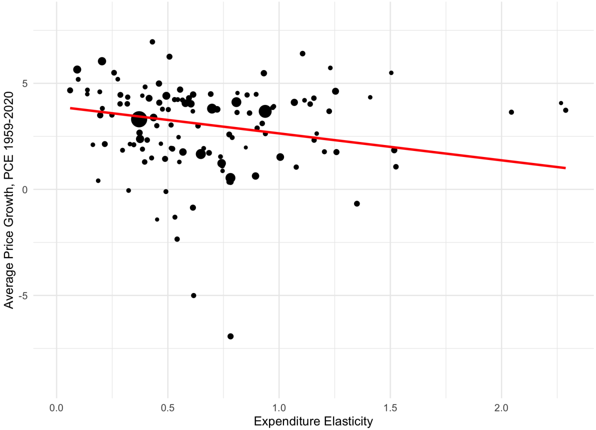
Income-elastic sectors experience lower inflation

Average Price Growth by PCE Category$_\varepsilon = \beta_0 + \beta_1\ \eta_\varepsilon$						
	Dep. Var.: Ave. Annual Price Growth in %					
	Ave. 1959-2020			Ave. 1980-2020		
	(1)	(2)	(3)	(4)	(5)	(6)
Exp. Elasticity η_ε	-1.20	-2.10	-1.67	-1.20	-1.82	-1.38
ε indexes PCE categories	(0.65)	(0.56)	(0.64)	(0.71)	(0.41)	(0.46)
Goods vs. Servs FE	N	Y	Y	N	Y	Y
2-digit Prod. Cat. FE	N	N	Y	N	N	Y
(partial) R^2	0.05	0.2	0.11	0.05	0.18	0.09

Not
exp

- Magnitude using IQR: $-1.2 \cdot (0.85 - 0.36) = -0.59\%$ differential.

Income-elastic sectors experience lower inflation



▸ Goods vs Services

Home Market Effects and the Linder Hypothesis

Central Result in Trade

- Classic paper in HME Krugman (1980).
- Linder Hypothesis: Countries specialize in the goods that they tend to consume more.
- Both go together well, consider particular case of nonhomotheticities: rich countries specialize in more income elastic goods and thus export them (see Fajgelbaum Grossman Helpman, Matsuyama 2019)
- Study this in two-country model static version of previous model

Static Version of the previous model

- Consider model without knowledge spillovers (Krugman 80).
- Substitute
$$\dot{N}\epsilon, t = \eta N_t^{\phi} L_{R\epsilon,t}$$
with a static model
$$N_{\epsilon,t} = \eta L_{R\epsilon,t}.$$
- Same qualitative predictions but no growth.
- Interpret lower nest as intermediate inputs to produce good ϵ.
  - Recall price $P_\epsilon \propto WN^{\frac{1}{1-\sigma}}$

Introduce trade

- Consider now two symmetric countries without trade costs.
  - As Krugman, suppose no overlap in varieties, two-way flow.
- Consider now economy w/ iceberg transportation cost $\tau > 1$.
  - Price charged at home is P_ϵ and abroad τP_ϵ.
  - Larger sales at home than abroad$\rightarrow$ home market is more important for domestic firms

Matsuyama (2019)

- Consider now two countries 1 and 2 with equal population sizes.
- Each country is populated by homogeneous agents who supply inelastically h^j of human capital, $h_1 > h_2$.
- Each agent has a nhCES demand as in the previous model.
- Demand of good ϵ is given by the sum of home and foreign

$$P_\epsilon^{-\rho}\left(E_1^\rho C_1^{(1-\rho)\epsilon} + \frac{1}{\tau} E_2^\rho C_2^{(1-\rho)\epsilon}\right)$$

  Recall that $P_\epsilon(N_\epsilon)$.
- Employment distribution in each country is no longer proportional to the market size distribution in that country
- Where do we have relatively more entry?

Home Market Effect: Equilibrium Properties

- Can show that there is a unique $\bar{\epsilon}$ cutoff above which there is relatively more labor employed in income elastic sectors in 1.
- The reason is that home demand in 1 is more skewed towards income elastic products.
- This is in **strong contrast** to a closed economy in which labor is allocated proportionally to the home market size.
- Thus, international trade magnifies the power of the domestic demand composition in dictating the allocation of resources across sectors!
- Country 1 becomes net exporter of income elastic products (Linder effect).
- In Krugman (1980) initial taste differences are given exogenously across countries, here depend on income effects.

A note on country size

- Note that we have increasing returns. So adding different country sizes may matter for understanding welfare effects and pattern of specialization.
- In this model it is possible for a large country with low h to be better off in autarky and in this case opening to trade would imply that it specializes in exporting income elastic goods!